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Applicable Method to Specular Surface for Recovering Sign of Local Gaussian Curvature

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Abstract

This paper proposes a new method to recover the sign of local Gaussian curvature from multiple shading images (more than three). The required information to recover the sign of Gaussian curvature is obtained by applying Principal Components Analysis (PCA) to the normalized irradiance measurements. The sign of the Gaussian curvature is recovered based on the relative orientation of measurements obtained on a local five point test pattern to those in the 2-D subspace, called the eigen plane. Using multiple shading images gives more correct and robust result and minimizes the effect of shadows by allowing a larger area of visible surface to be analyzed in comparison with the methods using the three shading images. Furthermore, it makes this method be applicable to the specular surface object where it is impossible for the previous method to recover the sign of the Gaussian curvature. On the other hand, since PCA removes a high degree of correlation between each image, this method can keep result high quality even when the light source directions are not widely dispersed.

1 Introduction

Sign of Gaussian curvature is a useful local descriptor of 3-D object shape since it is viewpoint invariant. It can be useful for tasks such as pose determination and segmentation. Some recent papers [1]-[3] describe methods to recover the sign of Gaussian curvature from three shading images acquired under different conditions of illumination.

This paper proposes a new method to recover the sign of local Gaussian curvature directly from multiple shading images (more than three) taken under different conditions of illumination. The method uses the information obtained by applying Principal Components Analysis (PCA) to the normalized irradiance measurements.

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Using more than three shading images gives more robust result than the previous methods and minimizes the effect of shadows by allowing a larger area of visible surface to be analyzed. Furthermore, it makes this method be applicable to the specular surface object by selecting the images for each point, assuming surface reflectance to be generic diffuse reflectance.

2 Three-Dimensional Space of Image Irradiances

In the three light sources case, the three irradiance measurements obtained at each pixel are denoted by (E_1, E_2, E_3), where E_1, E_2 and E_3 are considered to define the axes of a 3-D right-handed coordinate system. For a Lambertian surface with constant albedo, Woodham [4] showed that scatter plot of measurements, (E_1, E_2, E_3), define a 6-degree-of-freedom ellipsoid. This ellipsoid does not depend on the shape of the object in view nor on the relative orientation between object and viewer. Angelopoulou [3] showed that scatter plots for a variety of diffuse surfaces with constant albedo, including surfaces with varying degrees of surface roughness, remain ellipsoid-like in that they have positive Gaussian curvature everywhere.

Angelopoulou [3] also showed that the scatter plot for a surface with multiple distinct albedo gives multiple distinct ellipsoid-like shapes that differ only in scale. Following [3], we use normalization to remove the effect of varying albedo. Let $\boldsymbol{E}' = (E_1/\|\boldsymbol{E}\|, E_2/\|\boldsymbol{E}\|, E_3/\|\boldsymbol{E}\|)$. Then, the scatter plot of $\boldsymbol{E}'$ values produces a normalized shape of the ellipsoid-like plots in (E_1, E_2, E_3) space. Normalization, as defined here, extends in the obvious way to the p-dimensional case.

3 Recovering Sign of Gaussian Curvature from 2-D Subspace

3.1 2-D Subspace of Space of Normalized Image Irradiances

Let $\boldsymbol{\Upsilon}$ be the standard mapping from the unit surface normal at a point on a smooth object to the as-

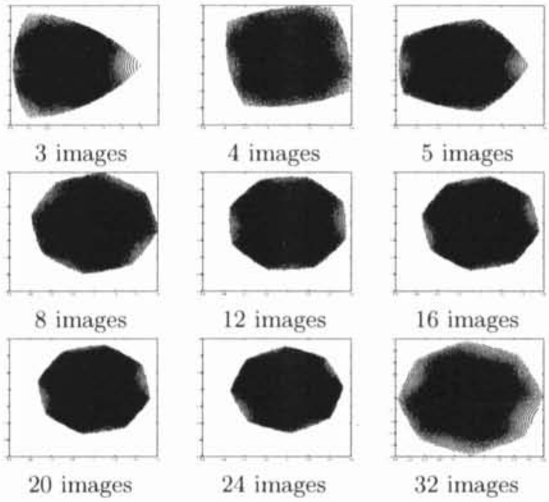


Figure 1. Scatter Plots on Eigen Plane for Lambertian Sphere

sociated point on the Gaussian sphere. The sign of local Gaussian curvature and the location of the points mapped by $\boldsymbol{\Upsilon}$ correspond each other.

For given conditions of illumination, let $\boldsymbol{\Phi}$ be the mapping from a point on the Gaussian sphere to the p-dimensional space of normalized irradiance measurements. For suitably illuminated points, $\boldsymbol{\Phi}$ is invertible since the p-tuple of image irradiances is different for each different surface normal.

The novel idea is to use Principal Components Analysis (PCA) to reduce the dimensionality of the space of measurements. Each point in the p-dimensional space of the normalized irradiances is mapped into the 2-dimensional subspace by a transformation denoted by $\boldsymbol{\Psi}$. $\boldsymbol{\Psi}$ selects the first two principal components of the original measurements. We call this 2-dimensional subspace the eigen plane. It is also noted that PCA is a mapping that can reduce the dimensionality of space with keeping the regularity of points in the space as much as possible. Examples of scatter plot on the eigen plane for a Lambertian sphere with different number of images are shown in Figure 1. It is shown that the regularity of points on the Gaussian sphere is preserved well on the eigen plane.

For the diffuse object that has all possible visible surface normal, the scatter plot on the eigen plane is similar to that shown in Figure 1 as far as the scatter plot in p-dimensional space of the normalized irradiances is similar to that for the sphere. While for the object that does not have all possible surface gradients, the scatter plot on the eigen plane becomes a part of that shown in Figure 1, and the density of the scatter plot corresponds to that of the surface gradient distribution. In that sense, to use the plot on the eigen plane is sufficient to recover the sign of Gaussian curvature of the object.

3.2 Two Types of Mapping

Consider the special case that a test object is a sphere. Define a image template of local five points consisting of a center point and top, bottom, left and right neighbors. Label the five points as ⓪ for the center point, ① for the top neighbor, ②, ③ and ④ for the left, bottom and right neighbors respectively (in counter-clockwise order). The corresponding points on the eigen plane will appear either in counter-clockwise or clockwise order. They will appear in the original labelling order if the mapping $\boldsymbol{\Psi} \circ \boldsymbol{\Phi}$ preserves the ordering of points on the sphere. The ordering of points on the eigen plane depends both on the light sources arrangement and on $\boldsymbol{\Psi}$.

All coordinate systems are assumed to be the right-handed coordinate systems. To argue the factor of light sources arrangement, three light source case is considered first. As far as the three light sources are arranged in counter-clockwise order with respect to the viewing direction, the local points in (E_1, E_2, E_3) space are preserved in the original labelling order[3]. The preservation or reversal of the ordering of points depends explicitly on the ordering of the light sources arrangement with respect to the viewing direction.

With p light sources, the ordering of the corresponding points on the eigen plane also depends on the light sources arrangement. Without loss of generality, assume that the light sources are given in counter-clockwise order with respect to the viewing direction (so that discussion about reversals owing to light sources ordering can be avoided in the following discussion).

Except the above condition, the mapping $\boldsymbol{\Psi}$ still may or may not preserve the ordering of the points ① to ④ when they are mapped onto the eigen plane.

$\boldsymbol{\Psi}$ consists of $(\boldsymbol{\psi}_1, \boldsymbol{\psi}_2)$ that are the eigen vectors of covariance matrix. Actually, four combinations exist for the directions of $\boldsymbol{\psi}_1$ and $\boldsymbol{\psi}_2$. This is based on the fact that $\boldsymbol{\psi}_i$ and $-\boldsymbol{\psi}_i (i = 1, 2)$ are the possible candidates and that preservation or reversal of the labelling order depends on the combinations of the directions of $\boldsymbol{\psi}_1$ and $\boldsymbol{\psi}_2$. However, when the eigen plane is defined as 2-D right-handed coordinate system, $\boldsymbol{\Psi}$ results in either of the only two types of mapping.

When $\boldsymbol{\Psi}$ preserves the ordering of the points ① to ④, we call it a "preservation mapping". When $\boldsymbol{\Psi}$ reverses the ordering, we call it a "reversal mapping".

For a given imaging situation, it is simple to test whether $\boldsymbol{\Psi}$ defines a preservation or a reversal mapping. Let $\boldsymbol{e}_1, \boldsymbol{e}_2, \cdots, \boldsymbol{e}_p$ be $(1, 0, \cdots, 0)^T, (0, 1, 0, \cdots, 0)^T, \cdots, (0, 0, \cdots, 1)^T$ respectively. Suppose $\boldsymbol{\Psi}$ maps $\boldsymbol{e}_1, \boldsymbol{e}_2, \cdots, \boldsymbol{e}_p$ to $\boldsymbol{e}_1', \boldsymbol{e}_2', \cdots, \boldsymbol{e}_p'$ respectively. The distribution of $\boldsymbol{e}_1', \boldsymbol{e}_2', \cdots, \boldsymbol{e}_p'$ determines whether $\boldsymbol{\Psi}$ is a preservation or a reversal mapping. $\boldsymbol{\Psi}$ becomes a preservation mapping if $\boldsymbol{e}_1', \boldsymbol{e}_2', \cdots, \boldsymbol{e}_p'$ appear in counter-clockwise order. Conversely, $\boldsymbol{\Psi}$ becomes a reversal mapping if they appear in clockwise order. [ASIDE: if the light sources are given in clockwise order then the sense is simply reversed. That is, $\boldsymbol{\Psi}$ is preservation if $\boldsymbol{e}_1', \boldsymbol{e}_2', \cdots, \boldsymbol{e}_p'$ appear in clockwise order and reversal if they appear in counter-clockwise

Table 1. How to Determine Sign of Gaussian Curvature from Distribution of Local Points on Eigen Plane

ordering on the eigen plane	Ψ	
	preservation	reversal
counter-clockwise	$G > 0$	$G < 0$
line or a point	$G = 0$	$G = 0$
clockwise	$G < 0$	$G > 0$

order]

After Ψ is determined as a preservation or reversal mapping, the sign of the Gaussian curvature is recovered from the distribution of five points on the eigen plane.

3.3 Procedure

Table 1 shows how to recover the sign of the Gaussian curvature from the pattern of local test points on the eigen plane. As before, let the five local points on the image be labeled ⓪ for the center point, ① for its upper point, ②, ③ and ④ for the other three points oriented counter-clockwise. Suppose Ψ is a preservation mapping. If ① to ④ are mapped onto the eigen plane in a counter-clockwise manner then $G > 0$. If ① to ④ are mapped onto the eigen plane in a clockwise manner then $G < 0$. While, if Ψ is a reversal, the sign is determined as simply reversed result. Regardless of whether Ψ is preservation or reversal, if ① to ④ are mapped to a line or a point on the eigen plane then $G = 0$.

4 Experiments

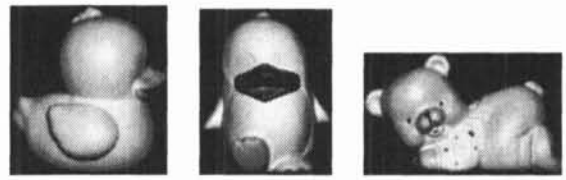
4.1 Diffuse Surface Case

Three objects (which have varying albedo) are tested. Images are acquired for two different zenith angles of illumination, eight with a zenith angle of 12[deg] and seven with a zenith angle of 17[deg]. Image examples of three objects are shown in Figure 2.

The experimental results are shown in Figure 3. Varying albedo is handled correctly. The theoretically correct result is not known. But, qualitatively the estimated sign of Gaussian curvature appears both correct and robust though the light source directions are not widely dispersed. The method works for almost the entire visible surface. Some points which look like noise in Figure 3 are misjudged. Image irradeances obtained around points of zero Gaussian curvature are mapped to nearby locations on the eigen plane. This sometimes causes the method to misjudge the sign.

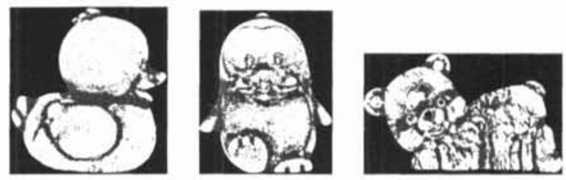
4.2 Non-Diffuse Surface Case

A glass with white color painting is used as a test object. It has the glossy surface reflectance. Measurement conditions are the same manner as those for the examples in Figure 2. Image samples are shown in Figure 4.



(a) (b) (c)

Figure 2. Shading Images (a) Duck (b) Penguin and (c) Bear



(a) (b) (c)

Figure 3. Results (a) Duck (b) Penguin and (c) Bear

The regularity will be lost between the distribution on the eigen plane and the original distribution on Gaussian sphere for the non-diffused area including specularity. It means that only the method itself can not be applied to the object with the glossy reflectance. So, the proposed method selects the images to be used for each point of the test object among all images, i.e., the images which cause the specularity are not used, instead the images that include only the diffused components are used.

Assume the case of the constant albedo. Specified points which include the glossiness are observed with the higher level of the image irradiance compared to other points. The threshold Th is used to judge whether any point includes glossiness or not. It is determined by the observed image itself.

The pixel coordinate (i, j) and the irradiance measurement E of (i, j) are denoted by (i, j, E), where i, j and E are considered to define the axes of a 3-D right-handed coordinate system. After all points on the image are plotted in the space, Th is determined by checking the peaks of the scatter plot in the space. Th is determined as around 180 for this example. Points that have higher image irradiance than Th are extracted as those of the glossy area.

Next, only the images that have lower irradiance than Th are used to recover the sign of Gaussian curvature at each point in the glossy area.

The results are shown in Figure 5-(a) and Figure 5-(b). In comparison with Figure 5-(a) and Figure 5-(b), it is obvious that removing the effect of glossy points among images can get better result in the glossy area. This extension is another advantage for that the glossiness causes other methods [2][3] to fail.

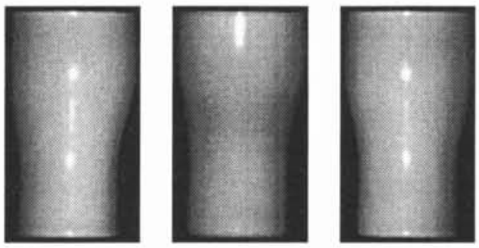
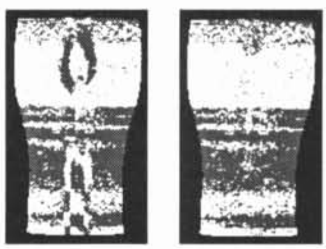


Figure 4. Example of Input Images



(a) (b)

Figure 5. (a)Result without Removal of Glossiness and (b) Result with Removal of Glossiness

For the evaluation, the observed image irradiances for this glass object are directly fitted to the Phong reflectance function shown in Eq. (1) and the parameters s and n are estimated.

$$E = C\{s\cdot(2cos(i)cos(e)-cos(g))^n+(1-s)cos(i)\} \quad (1)$$

where i, e and g are incidence angle, emittance angle and phase angle respectively, while the parameter n represents the width (i.e. sharpness) of the specular peak, the parameter s represents the percentage of the strength of the specular component to that of the diffuse component, and the parameter C is related to the albedo. The results are 0.18 for s and 43 for n, respectively.

Also, 2-D sinc function surface is used to evaluate the accuracy by the simulation. Images are synthesized under the conditions of s=0.18 and n=43. Measurement conditions are the same as those for the examples in Figure 4. The accuracy for this example is 96.7 %.

Next, we investigate the effectiveness by taking the various combinations of the parameters s and n. We use fifteen images of 2-D sinc function synthesized by the Phong reflectance function.

From the simulation, it is shown that the method can keep the accuracy of 90% when the value of parameter s takes 0.3 or less. Here, the glossy component becomes larger when the value of s becomes larger. Also the method is not sensitive to the change of the parameter n. Since the method still assumes the diffuse surface, the accuracy of the method becomes worse when s increases.

Although we used the Phong model to evaluate the limitation of the method, it should be noted that this method does not assume any specific model of diffuse surface reflectance.

5 Conclusion

This paper described a new method to recover the sign of local Gaussian curvature directly from multiple shading images. Generic diffuse reflectance is assumed. Principal components analysis is used to reduce a high dimensional problem to one of only two dimensions.

The sign of Gaussian curvature is obtained by comparing the relative orientation of five local test points in the image to that of the same points mapped onto the 2-D eigen plane. This is accomplished without any specific model of diffuse surface reflectance or specific information about the direction of the light sources.

Previous approaches used three light sources. Here, a larger number of light sources (and therefore a larger number of images) are used. Increased accuracy and robustness have been demonstrated, even when the light source directions are not widely dispersed. Spatially varying albedo also is handled correctly.

It is also demonstrated that the method is applicable to the glossy surface by selecting images for each point. Also the effective limit is given by simulation.

As the further subjects, the approach to enlarge the effective range of this method, or the approach to recover not only the sign but also its magnitude directly from multiple shading images, are remained.

Ackowlegement

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