# Components Recognition by the Generalized Hough Transform Using Multiple Two-dimensional Parameter Spaces 

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#### Abstract

The generalized Hough transform is a method for extracting similar figures to the template figure in an image. The generalized Hough transform needs huge memory for the fourdimensional parameter space which represents positions, scales and rotation angles of objective figures and requires processor power for conversing addresses from an image space into the parameter space. The paper proposes a method using multiple two dimensional parameter spaces for reducing the volume of the parameter space and for performing high speed processing. As the experimental result, the method provided high rate repeatability on the position, the scale and the rotation angle of extracted figures and performed short processing time.


## 1 Introduction

For automating assembly processes by machine vision technologies, measurement of positions, sizes and rotation angles of objects in an image is often required. The normalized correlation matching algorithm[1] is used for industrial image processing as the general-purposed method. However, this method is difficult to be applied to matching process when objects have different sizes and rotation angles from a template.
The generalized Hough transform[2] is known as a method which can be applied to different sizes and rotation angles. The generalized Hough transform is a robust method because objects are extracted by voted results into a parameter space which represents poses of objects. On the other hand, the generalized Hough transform needs a huge parameter space for voting process and much computational cost for transforming process. A method by hierarchical matching[3], a method by randomized matching[4] and a method by the separated parameter spaces[5][6] were proposed for the efficient matching process. In this paper, by substituting a four-dimensional parameter space that is necessary for the conventional generalized Hough transform for multiple twodimensional parameter spaces, a method for measuring positions, sizes and rotation angles of objects in an image is proposed. This method purposes to measure poses of industrial components mainly.

## 2 The generalized Hough transform

As shown in Fig.1, in the template registration phase in the generalized Hough transform, a base point $\mathrm{O}\left(x_{o}, y_{o}\right)$ is set in the template figure. About each edge point $\mathrm{P}_{1}\left(x_{i}, y_{i}\right)$ on the template figure, the edge direction angle $\theta_{i}$, the length $r_{i}$ and the angle $\alpha_{\text {, of the segment } \mathrm{P}, \mathrm{O} \text { are determined as geometrical }}^{\text {the }}$

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Fig. 1 Geometric parameters for the generalized Hough transform
parameters. Then, the template table is made as $\theta$, is a key and $r_{i}$ and $\alpha_{i}$ are data.
In the matching phase, the range and the resolution of positions $(x, y)$, sizes S and rotation angles $\phi$ of objective figures as against the template figure are set. The fourdimensional parameter space that is composed of these dispersed four kinds of parameters is prepared. When the figure with the scale $S_{u}$ and the rotation angle $\phi_{v}$ is the object for extracting, the edge direction angle $\theta_{j}$ is determined in an edge point $\mathrm{Q},\left(x_{j} y_{j}\right)$ in an image, and $\theta_{\text {, that }}$ is near to the value of $\theta_{j}-\phi_{v}$ is searched in keys of the template table. By using the value of $\mathrm{r}_{i}$ and $\alpha$, that correspond with $\theta_{i}$, the candidate coordinates $\left(x_{c}, y_{c}\right)$ for the base point of an object figure is determined by following expressions.

$$
\begin{align*}
& x_{c}=x_{j}+r_{i} \cdot S_{u} \cdot \cos \left(\alpha_{i}+\phi_{v}\right)  \tag{1}\\
& y_{c}=y_{j}+r_{i} \cdot S_{u} \cdot \sin \left(\alpha_{i}+\phi_{v}\right)
\end{align*}
$$

Then, voting process for increasing a value stored in the coordinates $\left(x_{c}, y_{c}, S_{u}, \phi_{v}\right)$ in the four-dimensional parameter space is executed. Calculating candidate coordinates for the base point and voting process to coordinates are executed about all dispersed values of $S$ and $\phi$. After voting process for all edge points in an image is completed, coordinates which have many accumulated votes express positions, scales and rotation angles of similar figures to the template figure. In case of ranges of positions of objects are broad and a high precise position is required, the generalized Hough transform needs a huge four-dimensional parameter space.

## 3 Template registration phase

This chapter describes about the template registration phase. First, each edge point that composes the template figure is extracted. Differential values $\Delta x_{i}$ to the x -direction and $\Delta y_{i}$ to the $y$-direction on a pixel $\mathrm{P}_{i}\left(x_{i} y_{i}\right)$ are obtained by the Prewitt operator, and the edge intensity $E_{i}$ is calculated by the next expression.

$$
\begin{equation*}
E_{i}=\left(\Delta x_{i}^{2}+\Delta y_{i}^{2}\right)^{1 / 2} \tag{3}
\end{equation*}
$$

| Edge points | $\theta_{1}$ | $r_{1}$ | $\alpha_{i}$ |
| :---: | :---: | :---: | :---: |
| 1 | $\theta_{1}$ | $r_{1}$ | $\alpha_{1}$ |
| 2 | $\theta_{2}$ | $r_{2}$ | $\alpha_{2}$ |
| $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ |
| $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ |
| $i$ | $\theta_{i}$ | $r_{1}$ | $\alpha_{1}$ |
| $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ |
| $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ |
| $N$ | $\theta_{N}$ | $r_{N}$ | $\alpha_{N}$ |

Fig. 2 Template table


Fig. 3 Parameter space configuration
When the edge intensity $E_{i}$ is more than a constant, this pixel is considered as the edge point, and the edge direction angle $\theta_{i}$ is calculated by the next expression.

$$
\begin{equation*}
\theta_{i}=\tan ^{-1}\left(\Delta y_{i} / \Delta x_{i}\right) \tag{4}
\end{equation*}
$$

And, geometrical parameters $r_{i}$ and $\alpha_{i}$ shown in Fig. 1 are determined. These geometrical parameters are determined on all edge points. The set of $\theta_{i}, r_{i}$ and $\alpha_{i}$ is regarded as an one element in the template table shown in Fig.2. When the template figure has $N$ edge points, the number of elements in the template table is also $N$.

## 4 Matching phase

### 4.1 Configuration of two-dimensional parameter spaces

All parameter spaces used for this method are twodimension. As it is shown in Fig.3, each parameter space has $512 \times 480$ pixels corresponding each other. The space for voting candidates for base points is used for storing accumulated votes to the candidate coordinates for base points of objective figures, and each coordinates has 16 bits depth. The space for storing rotation angles is used for storing sine values and cosine values of rotation angles of objective figures, and each coordinates has 16 bits depth. The space for storing previous rotation angles is used for storing sine values and cosine values of rotation angles of objective figures at the previous voting process, and each coordinates has 16 bits. This space purposes to evaluate changes of sine values and cosine values of rotation angles. The space for storing scale values is used for storing scales of objective figures, and each coordinates has 16 bits depth.

### 4.2 Calculating candidate coordinates for base points

As shown in Fig.4, it is supposed that an objective figure has


Fig. 4 Candidates of the base point
a scale $S$ and a rotation angle $\phi$ as against the template figure. And, it is supposed that the point $\mathrm{P}_{j}$ is corresponded to the point $P_{i}$ on the template figure. The edge direction angle $\theta_{j}$ at the point $\mathrm{P}_{j}$ is represented by the following expression.

$$
\begin{equation*}
\theta_{j}=\theta_{1}+\phi \tag{5}
\end{equation*}
$$

And, the direction angle $\beta_{j}$ to the candidate for the base point from the point $P_{j}$ is represented by the following expression.

$$
\begin{equation*}
\beta_{j}=\alpha_{i}+\phi \tag{6}
\end{equation*}
$$

From expression(5) and expression(6), $\beta_{j}$ is represented by the following expression.

$$
\begin{equation*}
\beta_{j}=\alpha_{i}+\theta_{j}-\theta_{i} \tag{7}
\end{equation*}
$$

Then, it is supposed that the maximum scale of objective figures is $S_{\max }$ and the minimum is $S_{\min }$. The candidate coordinates $\mathrm{O}_{s}\left(x_{s}, y_{s}\right)$ for the base point is calculated by the following expressions using $\theta_{i}, \alpha_{i}$ and $r_{i}$ in the template table when the scale is $S_{\min n}$.

$$
\begin{align*}
& x_{s}=x_{j}+r_{i} \cdot S_{\min } \cdot \cos \left(\alpha_{i}+\theta_{j}-\theta_{i}\right)  \tag{8}\\
& y_{s}=y_{j}+r_{i} \cdot S_{\min } \cdot \sin \left(\alpha_{i}+\theta_{j}-\theta_{i}\right) \tag{9}
\end{align*}
$$

The candidate coordinates $\mathrm{O}_{\mathrm{e}}\left(x_{e} y_{e}\right)$ for the base point is calculated by the following expressions when the scale is $S_{\text {max }}$.

$$
\begin{align*}
& x_{e}=x_{j}+r_{i} \cdot S_{\max } \cdot \cos \left(\alpha_{i}+\theta_{j} \cdot \theta_{i}\right)  \tag{10}\\
& y_{e}=y_{j}+r_{i} \cdot S_{\max } \cdot \sin \left(\alpha_{i}+\theta_{j}-\theta_{i}\right) \tag{11}
\end{align*}
$$

Because the true base point exists on the segment that joints the point $\mathrm{O}_{s}\left(x_{s} y_{s}\right)$ and the point $\mathrm{O}_{e}\left(x_{e} y_{e}\right)$, all points on the segment $\mathrm{O}_{s} \mathrm{O}_{\mathrm{c}}$ in the space for voting candidates for base points are candidates for voting process described in the section 4.4 .

### 4.3 Storing rotation angles

The sine value and the cosine value of $\phi=\theta, \theta$, that represents the rotation angle of an object figure are stored in each coordinates $(x, y)$ on the segment $\mathrm{O}_{s} \mathrm{O}_{\mathrm{e}}$ in the space for voting candidates for base points. Both the averaged sine values of all sine values and the averaged cosine values of all cosine values which are addressed to the coordinates $(x, y)$ previously are stored into the coordinates $(x, y)$. In the Hough transform, some noisy votes may be included in accumulated votes. Because the number of votes are accumulated by one in the space for voting candidates for base points, noisy votes do not influence the resulted votes relatively for the purpose of extracting the base point. But the purpose for determining the rotation angle, because the averaged value is renewed using noisy values, even if the number of noisy values is few, the resulted error increases. In this method, noisy values are removed using only two-dimensional spaces by the following processes.
First, a difference between a sine value which is addressed to the coordinates $(x, y)$ and a previous $\sin$ value which is stored in the spaces for storing previous rotation angle is obtained. The process is also executed about a cosine value. And, when these difference which are less than constants appear $R$ times continuously, an averaged sine value and an averaged cosine


Fgg 5 Extraction of data for roxation anges and change of accumulated votes
values of the previous number of $R$ sine values and $R$ cosine values are calculated. These averaged sine value and cosine value are set as the standard values for selecting following rotation angles. Only sine values and cosine values that are close to the standard values are selected after the standard values are set. Selected sine value and cosine value are added to the accumulated sine value and the accumulated cosine value which are addressed to the coordinates $(x, y)$ previously, and the averaged sine value and the averaged cosine value are calculated. And then, the new averaged sine value and the new averaged cosine value are stored into the coordinates $(x, y)$ in the space for storing rotation angles. These averaged values are used as the new standard values for selecting following sine values and cosine values. Therefore, the averaged sine value of all selected sine values and the averaged cosine value of all selected cosine values are stored in the space for storing rotation angles. Fig.5(a) shows a state for extracting rotation angles in case $R=8$. The number of eight data shown in white circles which are close to previous data are extracted. The averaged value of these eight values is set as the standard value for selecting following data. Only data shown in black circles in the shadow area that are close to the standard value are selected after setting the standard value. The averaged value which means the standard value is calculated whenever a new data is selected. On the other hand, data shown in squares that greatly differ from the standard value are ignored as noisy values.

### 4.4 Voting to candidates for the base point

Only when the value in the coordinates $(x, y)$ in the space for storing rotation angles is changed, the value in the coordinates $(x, y)$ in the space for voting candidates for base points is increased by one as voting process. Fig.5(b) shows a transition of accumulated votes. Before the standard value for selecting rotation angles data is set, that is until continuous eight data appear in this sample, the number of votes is initialized to one whenever a data which has large difference between a previous data appears. The number of votes is accumulated after the standard value is set.

### 4.5 Storing scales

Only when the value in the coordinates $(x, y)$ in the space for storing rotation angles is changed, the scale S is stored into the coordinates $(x, y)$ in the space for storing scale values. Because the coordinates $(x, y)$ means the internal divided point
on the segment $\mathrm{O}_{s} \mathrm{O}_{e}$ which combines the point $\mathrm{O}_{s}$ corresponding to the scale $S_{m i n}$ and the point $\mathrm{O}_{\mathrm{e}}$ corresponding to the scale $S_{\max }$, the scale S is calculated by the following expression.

$$
S=\left\{\left(x-x_{s}\right)^{2}+\left(y-y_{s}\right)^{2}\right\}^{12}\left(S_{\max }-S_{\min }\right) /\left\{\left(x_{e}-x_{s}\right)^{2}+\left(y_{e}-y_{s}\right)^{2}\right\}^{1 / 2}+S_{\min } \text { (12) }
$$

At the edge point $P$, shown in Fig.4, voting process and storing rotation angles and storing scales are executed using all geometric parameters in the template table. Then, processes described above are executed at all edge points other than the point $\mathrm{P}_{j}$.

### 4.6 Calculating coordinates of base points

Coordinates which have the accumulated votes more than a constant and their neighboring coordinates are extracted in the space for voting candidates for base points. Coordinates of base points of objective figures are calculated as gravitational centers using accumulated values in extracted coordinates.

### 4.7 Calculating rotation angles and scales

The same coordinates as extracted coordinates for calculating coordinates of base points in the section 4.6 are extracted in the space for storing rotation angles and in the space for storing scale values. The median in values in extracted coordinates in the space for storing rotation angles is made the


Fig. 6 A recognized image without noisy figures


Fig. 7 A recognized image with noisy figures

Table 1 Comparison of processing time

| Test images | Image <br> in Fig.6 <br> sec | Image <br> in Fig.7 <br> sec |
| :--- | ---: | ---: |
| Methods <br> Using four-dimensional parameter space | 128 | 288 |
| Proposed generalized Hough transform using <br> two-dimensional parameter spaces | 25 | 55 |

rotation angle of an objective figure. The median in values in extracted coordinates in the space for storing scale values is made the scale of an objective figure.

## 5 Experiments

### 5.1 Extracting figures

Fig. 6 shows extracted similar figures to the template figure in a sample image. The number of pixels is $512 \times 480$. The maximum scale is $S_{\text {max }}=1.0$ and the minimum scale is $S_{\min }=0.5$. The number of data for setting the standard value for storing rotation angles is $R=8$. The rectangle that encloses each extracted figure shows the scale, and the direction of the segment drawn from the base point of each figure represents rotation angle. And, Fig. 7 shows the resulted image when various kinds of figures are included in the image and some figures are overlapped and chipped.

### 5.2 Evaluating repeatability

The averaged position, the averaged scale and the averaged rotation angle were calculated using data of extracted five figures in Fig. 6 in 100 times processing. Then, the difference between each data in 100 times processing and the averaged data was evaluated as the repeatability. The repeatability of positions resulted $\pm 0.8$ pixels in both x and y directions. The repeatability of scales resulted $\pm 0.03$. The repeatability of rotation angles resulted $\pm 3$ degrees.

### 5.3 The amount of parameter spaces and processing time

About the total amount of parameter spaces and processing time, this method was compared with the conventional generalized Hough transform. When coordinates in the fourdimensional parameter space have 16 bits depth, the range of scales is from 0.5 to 1.0 by 0.06 and the range of rotation angles is from 0 to 360 degree by 6 degree, the total $480 \times$ $512 \times\{(1.0-0.5) / 0.06\} \times(360 / 6) \times 2 \fallingdotseq 265 \mathrm{MB}$ parameter space is required for realizing the precision that is almost equal to this method. On the other hand, this method needs only $480 \times 512 \times 6 \times 2=2.95 \mathrm{MB}$ space for six 16 bits twodimensional parameter spaces as shown in Fig. 3 This space is about $1 / 90$ of the four-dimensional parameter space.
Next, processing time for this method was compared with the conventional Hough transform using the four-dimensional parameter space whose amount is $480 \times 512 \times\{(1.0-0.5) / 0.1\}$ $\times(360 / 18) \times 2 \fallingdotseq 49.2 \mathrm{MB}$, because the resolution of scales is limited to 0.1 and the resolution of rotation angles is limited to 18 degree for the realizable experiment. The images used for the experiment were Fig. 6 and Fig.7. Pentium-133MHz was used for the processing system. As shown in Table 1, this method is about five times faster than the method which uses 49.2MB four dimensional parameter space. It is estimated that this method is about 27 times faster than the method which needs the 265 MB four-dimensional parameter spaces whose resolutions about scales and rotation angles are almost same as this method.


Fig. 8 Extracted metal fittings

### 5.4 Extracting metal fittings

Fig. 8 shows extracted metal fittings that have different sizes and are overlapped each other. The processing time was about 30 seconds for extracting metal fittings.

## 6 Conclusions

The generalized Hough transform using multiple twodimensional parameter spaces was proposed for extracting arbitrary shaped figures in an image. This method does not need the huge four-dimensional parameter space for the conventional generalized Hough transform because voting into coordinates for candidates of base points, storing scales and storing rotation angles are executed using only twodimensional parameter spaces. As the experimental result, repeatability about the position, the scale and the rotation angle reached each $\pm 0.8$ pixels, 0.25 and $\pm 3$ degree. And, the processing time was about $1 / 27$ times shorter than the conventional generalized Hough transform.

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