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3D SHAPE RECONSTRUCTION OF UNFOLDED BOOK SURFACE FROM A SCANNER IMAGE

— SHAPE FROM SHADING WITH INTERREFLECTIONS UNDER PROXIMAL LIGHT SOURCE —

Hiroyuki UKIDA, Toshikazu WADA and Takashi MATSUYAMA
Department of Information Technology, Faculty of Engineering, Okayama University
3-1-1, Tsushima Naka, Okayama-shi, Okayama 700, Japan

ABSTRACT

In this paper, we propose an efficient and stable algorithm to recover the 3D shape of an unfolded curved book surface from the shading information of a scanner image. This shape from shading problem in a real world environment has a global and complicated interdependency among the shape, the depth and the albedo distribution of the book surface. Hence, the overall surface shape must be recovered simultaneously. From a computational point of view, if we used pointwise descriptions of the shape and the albedo of the book surface, the recovering process would require a lot of time and memory, and moreover, the solution would not be stable against noise. To improve the efficiency and the stability, we propose a recovering algorithm which employs piecewise approximations of the shape and the albedo. Some experiments demonstrated that this algorithm drastically reduces the computation time to recover the book surface keeping the enough accuracy to restore the distorted scanner image of the 3D book surface.

1 INTRODUCTION

In this paper, we address the problem to recover the 3D shape of a book surface from the shading information of a scanner image. This shape from shading problem in a real world environment has the following characteristics:
1. Proximal light source: The light source is located close to the book surface. This implies that the illuminant intensity varies with the distance between the light source and the book surface, and the light source direction also varies with the point on the book surface.
2. Interreflections: The light reflected on the right (left) side of the book surface illuminates the left (right) side.
3. Moving light source: The light source moves while taking a scanner image.
4. Specular component of reflections: The book surface is not the Lambertian surface.
5. Nonuniform albedo distribution: The albedo distribution is not uniform on the book surface.

In the following sections, we formulate the shape from shading problem taking account of all these characteristics, and propose an efficient and stable algorithm to recover the 3D shape of the book surface.

2 PROBLEM FORMULATION

First, we describe the ideal shape from shading problem under the following conditions:
a. The light source is distant from the object surface. This implies that the illuminant intensity and the light source direction are constant on the object surface.
b. There are no interreflections.
c. The location of the light source is fixed.
d. The object has the Lambertian surface.
e. The albedo is constant allover the object surface.
The ideal problem under these conditions is formulated as:

$$I_o(\boldsymbol{x}) = \rho \cdot I_s \cdot \cos\varphi(\boldsymbol{x}), \tag{1}$$

where $\boldsymbol{x}$ is a 2D point of the image, $I_o(\boldsymbol{x})$ is the reflected light intensity observed at $\boldsymbol{x}$, I_s is the illuminant intensity, ρ is the albedo on the surface and $\varphi(\boldsymbol{x})$ is the angle between the light source direction and the surface normal at the point on the object surface corresponding to $\boldsymbol{x}$.

In this problem, it is obvious that
• $\varphi(\boldsymbol{x})$ can easily be calculated when $\rho \times I_s$ are given[1].
The shape of the object surface can be computed from $\varphi(\boldsymbol{x})$ and some additional constraints: photometric stereo[2] and shape constraints such as smoothness[3] and cylindrical surface[4].

Next, we briefly describe a stepwise formulation of the shape from shading problem under characteristics 1~5 described before.

Proximal light source(characteristic 1)

$$I_o(\boldsymbol{x}) = \rho \cdot I_s(d(\boldsymbol{s}(\boldsymbol{x}), \boldsymbol{l})) \cdot \cos\varphi(\boldsymbol{x}), \tag{2}$$

where $\boldsymbol{l}$ is the 3D location of the light source, $\boldsymbol{s}(\boldsymbol{x})$ is the 3D point on the object surface corresponding to $\boldsymbol{x}$ and $d(\boldsymbol{s}(\boldsymbol{x}), \boldsymbol{l})$ is the distance between $\boldsymbol{l}$ and $\boldsymbol{s}(\boldsymbol{x})$. We assume that ρ and $\boldsymbol{l}$ are given.
• In this problem, the absolute location (depth) of the object surface $\boldsymbol{s}(\boldsymbol{x})$ is required to compute $\varphi(\boldsymbol{x})$ at $\boldsymbol{x}$.

Interreflections[5][2](characteristic 2)

$$I_o(\boldsymbol{x}) = \rho \left\{ I_s + \int \frac{I_o(\boldsymbol{x}')}{\{d(\boldsymbol{s}(\boldsymbol{x}), \boldsymbol{s}(\boldsymbol{x}'))\}^2} d\boldsymbol{x}' \right\} \cos\varphi(\boldsymbol{x}), \tag{3}$$

• The global shape of the object surface ($\forall \boldsymbol{x}'$ $d(\boldsymbol{s}(\boldsymbol{x}),\boldsymbol{s}(\boldsymbol{x}'))$) is required to compute $\varphi(\boldsymbol{x})$.

Interreflections under proximal light source (characteristics 1 and 2)

$$I_o(\boldsymbol{x}) = \rho \left\{ I_s(d(\boldsymbol{s}(\boldsymbol{x}), \boldsymbol{l})) + \int \frac{I_o(\boldsymbol{x}')}{\{d(\boldsymbol{s}(\boldsymbol{x}), \boldsymbol{s}(\boldsymbol{x}'))\}^2} d\boldsymbol{x}' \right\} \times \cos\varphi(\boldsymbol{x}). \tag{4}$$

• The overall depth ($\forall \boldsymbol{x}$ $\boldsymbol{s}(\boldsymbol{x})$) of the object surface is required to compute $\varphi(\boldsymbol{x})$.

[1] We can determine $\rho \times I_s$ assuming that $\varphi(x)$ is equal to zero at x where $I_o(x)$ have the maximum value.

[2] In this formulation, we assume that 1) the light reflected at $s(x')$ can reach $s(x)$ for any x and x', 2) the light reflected more than once is enough attenuated to be neglected.

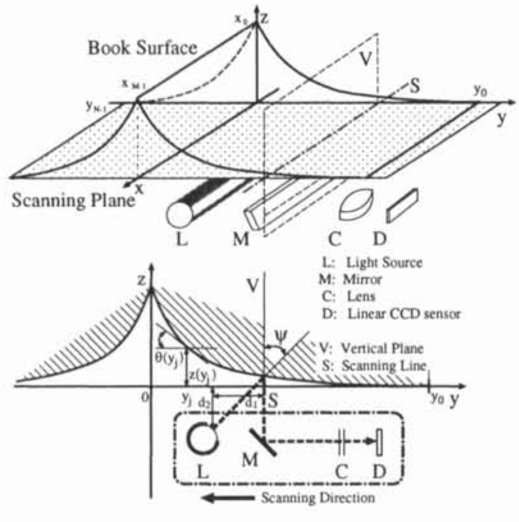

Figure 1: Structure of image scanner and book surface

Moving light source(characteristics 1~3)

$$I_o(\boldsymbol{x}) = \rho\left\{I_s(d(\boldsymbol{s}(\boldsymbol{x}), \boldsymbol{l}(\boldsymbol{x}))) + \int \frac{I_o'(\boldsymbol{s}(\boldsymbol{x}'), \boldsymbol{l}(\boldsymbol{x}))}{\{d(\boldsymbol{s}(\boldsymbol{x}), \boldsymbol{s}(\boldsymbol{x}'))\}^2}d\boldsymbol{x}'\right\} \times \cos\varphi(\boldsymbol{x}), \quad (5)$$

where $I_o'(\boldsymbol{s}(\boldsymbol{x}'), \boldsymbol{l}(\boldsymbol{x})) = \rho \cdot I_s(d(\boldsymbol{s}(\boldsymbol{x}'), \boldsymbol{l}(\boldsymbol{x}))) \cdot \cos\varphi(\boldsymbol{x}')$ and $\boldsymbol{l}(\boldsymbol{x})$ is the light source location corresponding to $\boldsymbol{x}$.

- $\forall \boldsymbol{x}$ $\boldsymbol{s}(\boldsymbol{x})$ are also required to compute $\varphi(\boldsymbol{x})$, but $I_o'(\boldsymbol{s}(\boldsymbol{x}'), \boldsymbol{l}(\boldsymbol{x}))$ must be calculated at each point on the object surface. Hence, the computation is more expensive than the problem under the fixed light source[5].

The addressed problem(characteristics 1~5)

$$I_o(\boldsymbol{x}) = \left\{I_s(d(\boldsymbol{s}(\boldsymbol{x}), \boldsymbol{l}(\boldsymbol{x}))) + \int \frac{I_o'(\boldsymbol{s}(\boldsymbol{x}'), \boldsymbol{l}(\boldsymbol{x}))}{\{d(\boldsymbol{s}(\boldsymbol{x}), \boldsymbol{s}(\boldsymbol{x}'))\}^2}d\boldsymbol{x}'\right\} \times \rho(\boldsymbol{s}(\boldsymbol{x})) \times f(\varphi(\boldsymbol{x}), \boldsymbol{s}(\boldsymbol{x})), \quad (6)$$

where $I_o'(\boldsymbol{s}(\boldsymbol{x}'), \boldsymbol{l}(\boldsymbol{x})) = \rho(\boldsymbol{s}(\boldsymbol{x}')) \cdot I_s(d(\boldsymbol{s}(\boldsymbol{x}'), \boldsymbol{l}(\boldsymbol{x}))) \cdot f(\varphi(\boldsymbol{x}'), \boldsymbol{s}(\boldsymbol{x}'))$, $\rho(\boldsymbol{s}(\boldsymbol{x}))$ is the albedo and $f(\varphi(\boldsymbol{x}), \boldsymbol{s}(\boldsymbol{x}))$ is the reflectance property at the point $\boldsymbol{s}(\boldsymbol{x})$.

- In this problem, $\forall \boldsymbol{x}$ $\boldsymbol{s}(\boldsymbol{x})$ and $\forall \boldsymbol{x}$ $\rho(\boldsymbol{s}(\boldsymbol{x}))$ are required to compute $\varphi(\boldsymbol{x})$. In other words, there is an interdependency among $\varphi(\boldsymbol{x})$, $\boldsymbol{s}(\boldsymbol{x})$ and $\rho(\boldsymbol{s}(\boldsymbol{x}))$.

3 PRACTICAL MODEL

3.1 Image Scanner and Book Surface

Figure 1 shows the structure of the image scanner and the coordinate system to describe the problem. The sensor D takes a 1D image $P^*(x_i)$ along the scanning line S and moves with L,M and C. The sequence of $P^*(x_i)$s forms a 2D image $P^*(x_i, y_j)$. While $P^*(x_i)$ is obtained by the perspective projection, the projection along the y-axis is equivalent to the orthogonal projection.

We formulate the problem under the following assumptions:

(1) The book surface is cylindrical and the shape of its cross section on the y-z plane is smooth except for the point separating the book pages.

(2) The unfolded book surface is aligned on the scanning plane so that the center line separating book pages is just above the x-axis.

These assumptions reduce the 3D shape reconstruction problem to the 2D cross section shape reconstruction problem.

3.2 Optical Model

The relationship among the image intensity and the reflected light intensity is formulated as follows:

$$P(x_i, y_j) = \alpha \cdot \rho(x_i, y_j)(I_{dir}(x_i, y_j) + I_{inter}(x_i, y_j)) + \beta, \quad (7)$$

where

- $P(x_i, y_j)$: The image intensity at (x_i, y_j) of the observed image.
- α, β: Parameters of the photoelectric transformation in the image scanner.
- $I_{dir}(x_i, y_j)$: The reflected light component corresponding to the direct illumination from the light source.
- $I_{inter}(x_i, y_j)$: The reflected light component corresponding to the indirect illumination from the opposite side of the book surface.

$I_{dir}(x_i, y_j)$ and $I_{inter}(x_i, y_j)$ are formulated as follows:

$$I_{dir}(x_i, y_j) = I_s(y_j, z(y_j), y_j) \cdot f(\boldsymbol{n}_1, \boldsymbol{l}_1, \boldsymbol{v}_1), \quad (8)$$

$$I_{inter}(x_i, y_j) = A \sum_{y_n=y_0}^{y_{N-1}} V(y_n, y_j) \cdot I_s(y_n, z(y_n), y_j) \times \sum_{x_m=x_0}^{x_{M-1}} \rho(x_m, y_n) \frac{f(\boldsymbol{n}_2, \boldsymbol{l}_2, \boldsymbol{v}_2) \cdot f(\boldsymbol{n}_1, \boldsymbol{l}_3, \boldsymbol{v}_1)}{\{d(x_m, y_n, x_i, y_j)\}^2}, \quad (9)$$

where

- $z(y_j)$: The distance between the scanning plane and the book surface (Figure 1). $z(y_j)$ is represented as follows:

$$z(y_j) = \sum_{y_k=y_0}^{y_j} \tan\theta(y_k) \quad (0 < y_j < y_0), \quad (10)$$

where $\theta(y_j)$ is the slant angle of the surface.

- $I_s(y, z, y_j)$: The illuminant intensity on the y-z plane when scanning the 1D image at y_j. Since the light source L can be approximated by the linear light source model, $I_s(y, z, y_j)$ is formulated as follows:

$$I_s(y, z, y_j) = \frac{I_D(\psi(y, z, y_j))}{\sqrt{(y - (y_j - d_1))^2 + (z + d_2)^2}} + I_e, \quad (11)$$

$$\psi(y, z, y_j) = \arctan\left(\frac{y - (y_j - d_1)}{z + d_2}\right), \quad (12)$$

where $(y_j - d_1, -d_2)$ is the location of the light source, $\psi(y, z, y_j)$ is the angle between the vertical line and the light source direction, $I_D(\psi)$ is the directional distribution of the illuminant intensity and I_e is the environment light intensity.

- $f(\boldsymbol{n}, \boldsymbol{l}, \boldsymbol{v})$: The reflectance property on the book surface. We employ the Phong's model[6] to represent both the diffuse and specular components of the reflected light.

$$f(\boldsymbol{n}, \boldsymbol{l}, \boldsymbol{v}) = s\cos\varphi(\boldsymbol{n}, \boldsymbol{l}) + (1 - s)\cos^n \delta(\boldsymbol{n}, \boldsymbol{l}, \boldsymbol{v}), \quad (13)$$

where $\boldsymbol{n}$ is the surface normal, $\boldsymbol{l}$ is the direction of the illumination and $\boldsymbol{v}$ is the direction of the view point. $\boldsymbol{n}$, $\boldsymbol{l}$, $\boldsymbol{v}$ are corresponding to $\boldsymbol{n}_1, \boldsymbol{l}_1, \boldsymbol{v}_1, \cdots$, etc. as shown in Figure 2. φ is the angle between $\boldsymbol{n}$ and $\boldsymbol{l}$, and δ is the angle between $\boldsymbol{v}$ and the direction of the specular reflection. s,n are the parameters to specify the reflectance property.

- A: The size of a pixel in the image.
- $V(y_n, y_j)$: The visibility function: If the reflected light at $(x_m, y_n, z(y_n))$ can reach $(x_i, y_j, z(y_j))$, then this function is 1. Otherwise it is 0.

In experiments, before recovering the book shape, parameters (α, β, d_1, d_2, I_e, s, n, $I_D(\psi)$) are estimated by images taking white slopes which have known slants.

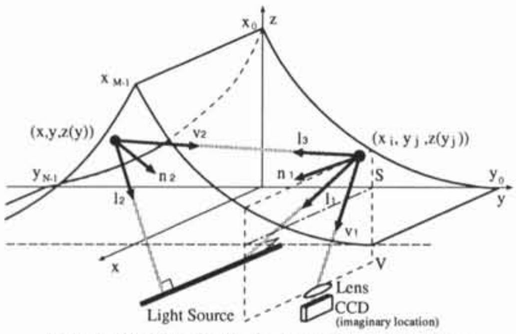

Figure 2: Interreflections on book surface

4 SHAPE RECONSTRUCTION

The problem is to estimate the shape information $\theta(y_j)$, the depth $z(y_j)$ and the albedo $\rho(x_i, y_j)$ which minimize the total error between the observed image intensity $P^*(x_i, y_j)$ and the calculated image intensity $P(x_i, y_j)$ by equation (7). The depth $z(y_j)$ can be calculated from $\theta(y_j)$ by equation (10), and the albedo $\rho(x_i, y_j)$ can be also calculated from $\theta(y_j)$, $z(y_j)$ and $P^*(x_i, y_j)$ by equation (7). Hence, the problem is essentially equivalent to finding optimal $\theta(y_j)$s which minimize the total error.

This problem is formalized as a non-linear least square problem in N-dimensional space, where N represents the number of sampling points along the y-axis (Figure 2). For the numerical solution of this problem, simultaneous equations with $N \times N$ sized coefficient matrix must be solved iteratively. But, in the practical problem, there are thousands of sampling points, and hence, the straightforward solution becomes computationally expensive. Moreover, a local noise of the image introduces an error into $\theta(y_j)$, which is accumulated by equation (10) and lead global errors of $z(y_j)$s.

4.1 Piecewise Shape and Albedo Approximation

To improve the computational efficiency and the stability, we employ the following piecewise approximations of the book surface:

1. Piecewise polynomial model fitting: a representation of the 2D cross section shape by m quadratic polynomials. The p-th polynomial is represented as follows:

$$Q_p(y) = \frac{z'_p - z'_{p-1}}{2(y_p^\Delta - y_{p-1}^\Delta)}(y - y_{p-1}^\Delta)^2 + z'_{p-1}(y - y_{p-1}^\Delta) + z_{p-1}, \tag{14}$$

where y_p^Δ $(p = 0, 1, \cdots, m)$ is the sampled point of y_j at uniform intervals of Δ pixels ($y_p^\Delta = y_{\Delta\times p}$), $z_p = z(y_p^\Delta)$, $z'_p = 2(z_p - z_{p-1})/(y_p^\Delta - y_{p-1}^\Delta) - z'_{p-1}$ and $z_0 = z'_0 = 0$ (just on the scanning plane). By using this model, the number of parameters to describe the cross section shape is reduced to m.

2. Tessellation of the book surface: an approximation of the 3D book surface by piecewise planar rectangles with constant albedos. By using this approximation, computation time of $I_{inter}(x_i, y_j)$ can be reduced.

4.2 Shape Recovering Algorithm

We propose the following iterative algorithm to recover the cross section shape of the book surface:

step 1 Estimate the initial shape by using the optical model ignoring $I_{inter}(x_i, y_j)$ in equation (7). In this estimation, the optimal number of polynomials m is also calculated based on the MDL criterion[7].

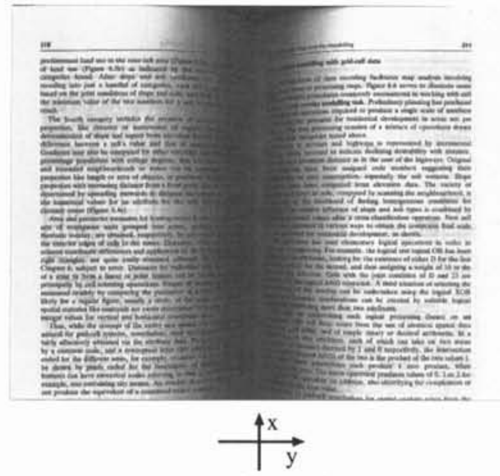

Figure 3: Observed image

step 2 Recover the albedo distribution by using the initial shape and the observed image $P^*(x_i, y_j)$.
step 3 Calculate $I_{inter}(x_i, y_j)$ by using the tessellated book surface.
step 4 Calculate all z_ps based on $I_{inter}(x_i, y_j)$ (step 3) and $P^*(x_i, y_j)$.
step 5 Recover the albedo distribution by the estimated shape (step 4), $I_{inter}(x_i, y_j)$ (step 3) and $P^*(x_i, y_j)$.
step 6 If all z_ps converge, then this algorithm is terminated. Otherwise repeat from step 3.

In step 4, all z_ps are calculated by the observed image intensity $P_w^*(y_j)$ at unprinted white background. $P_w^*(y_j)$ can be obtained from the observed image $P^*(x_i, y_j)$ as follows:

$$P_w^*(y_j) = \max_{x_i} P^*(x_i, y_j). \tag{15}$$

The optical model at the white background having the constant albedo ρ_w is represented as follows:

$$P_w(y_j) = \alpha \cdot \rho_w \cdot (I_{dir}(x(y_j), y_j) + I_{inter}(x(y_j), y_j)) + \beta, \tag{16}$$

where $x(y_j)$ is the location of the white background at y_j and we assume that ρ_w is given.

All z_ps are calculated by using method 1 followed by method 2.

method 1 Calculate z_p sequentially by minimizing the function G in each interval:

$$G(p, z_p) = \sum_{y_j = y_{p-1}^\Delta}^{y_p^\Delta} \{P_w^*(y_j) - P_w(y_j)\}^2. \tag{17}$$

method 2 More accurately, calculate all z_ps simultaneously by minimizing the function H. In this method, results of method 1 are used as initial estimates of z_ps.

$$H(z_1, \ldots, z_m) = \sum_{p=1}^{m} G(p, z_p). \tag{18}$$

Also in step 1, these methods are employed to estimate the initial shape by using the optical model ignoring $I_{inter}(x_i, y_j)$ in equation (16).

5 EXPERIMENTS

First, we show the experimental results of the shape estimation and the image restoration of a real book surface. Figure 3 shows an image of a book surface taken

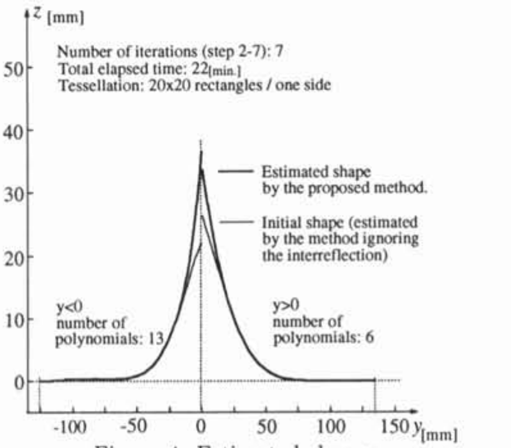

Figure 4: Estimated shapes

Table 1: Effectiveness of the piecewise polynomial model fitting

	number of parameters	ratio	time (real [min.])	error [mm]
piecewise	15	1	(1.18)	0.94
pointwise	480	233	(276.17)	1.28

by the scanner. Figure 4 shows the estimated cross section shapes. The thin line denotes the initial estimation and the bold line denotes the final estimation. Figure 5 shows the restored image using the estimated shape. The restored image is basically generated by rearranging the estimated albedos. For the fine image restoration, we use the following methods: 1. enhance the contrast between the albedos at unprinted area and the printed area, 2. use the cubic convolution for interpolating the albedos, 3. remove the shading along the x-axis caused by the limited length of the light source. It is confirmed that the readability of the book surface is considerably improved by the image restoration, and hence, the shape is accurately estimated enough for the image restoration.

Next, we show the experimental result to examine the effectiveness and the accuracy of the proposed algorithm using an artificial model having the known shape. Table 1 shows the computation time[3] to recover the shape and the mean error of the estimated shape by the piecewise polynomial model fitting and the pointwise method[4]. This result demonstrates that the piecewise shape approximation drastically reduces the computation time. Moreover, the accuracy of the estimation is improved, because the piecewise approximation is stable against noise. Table 2 shows the computation time and the mean error with $n \times n$ tessellation of the book surface. We can notice that the tessellation with adequate number of rectangles, such as $n = 20$, extremely accelerates the shape reconstruction process keeping the accuracy.

6 CONCLUSIONS

In this paper, we discussed the real world problem to recover the 3D shape of the book surface from a scanner image. It is shown that this problem is formalized as a non-linear least square problem to estimate the mutually depending parameters of the shape, the depth and the albedo. If we used the pointwise descriptions of these parameters, the shape reconstruction process would require a lot of time and memory, and the solution would not be stable against noise.

To improve the efficiency and the stability, we proposed a shape recovering algorithm which employs the piecewise approximations of the shape and the albedo distribution. Some experimental results demonstrated that the proposed algorithm can recover the 3D shape accurately and efficiently.

Problems under the following conditions are included in the future work: 1. the center line separating the book pages is not aligned parallel to the x-axis, 2. the reflectance property is unknown.

[3] We used a SPARC station 10 workstation for experiments.

[4] In this experiment, half of the book shape ($y > 0$) is recovered from the image without interreflections.

Figure 5: Restored image

Table 2: Effectiveness of the tessellated book surface

n	number of rectangles	time				error [mm]
		interreflections ratio (real [sec.])		reconstruction ratio (real [min.])		
1	2	0.0026	(0.07)	2.9	(1.3)	22.04
5	50	0.028	(0.76)	3.7	(1.7)	3.46
10	200	0.11	(2.90)	3.1	(1.5)	2.19
20	800	0.43	(11.8)	4.4	(2.1)	2.03
40	3,200	1.67	(46.2)	11.4	(5.4)	2.16
point-wise	628,145	100.0	(2735.5)	100.0	(47.1)	2.35

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