MVA'94 IAPR Workshop on Machine Vision Applications Dec. 13-15, 1994, Kawasaki

DIRECT RECONSTRUCTION OF PLANAR SURFACES BY STEREO VISION

Yasushi Kanazawa* and Kenichi Kanatani†

*Department of Information and Computer Engineering
Gunma College of Technology, Maebashi, Gunma 371, Japan
†Department of Computer Science
Gunma University, Kiryu, Gunma 376, Japan

Abstract

This paper studies the problem of reconstructing a planar surface by observing multiple feature points that are known to be coplanar in the scene. This paper presents a direct method for reconstructing a planar surface by applying the principle of maximum likelihood estimation based on geometric constraints and a statistical model of image noise. The significant fact about our method is that not only the 3-D position of the surface is reconstructed accurately but its reliability is also computed quantitatively.

1 Introduction

Stereo is one of the most fundamental means of 3-D sensing from images and is widely used as a visual sensor for autonomous navigation of robots [1, 7]. In the past, the study of stereo has mainly focused on the *correspondence detection* between the two images. However various other issues arise when we reconstruct 3-D from detected correspondences. First of all, the 3-D reconstruction should be accurate. Hence, we must maximize the accuracy by optimization techniques based on the statistical characteristic of image noise. At the same time, *the reliability of the reconstructed 3-D must be evaluated* [6]. If the errors involved in the reconstructed 3-D cannot be estimated, robots cannot take appropriate actions to archive given tasks effectively. This paper presents a new theory for reconstructing planar surfaces by stereo in a statistically optimal way and evaluating the reliability of the reconstruction in quantitative terms.

In order to reconstruct an optimal planar surface, we introduce the principle of *maximum likelihood estimation* and derive a scheme of nonlinear optimization for optimal estimations. At the same time, we derive a *theoretical lower bound* on the attainable accuracy of estimation. In order to compute the optimal solution, we use a numerical scheme called *renormalization* [3]. By numerical simulation, we show that the obtained solution almost attains the theoretical lower bound on accuracy. This means that we can quantitatively predict the reliability of the reconstructed surfaces. This has a great significance in robotics applications of stereo.

2 Camera and Noise Model

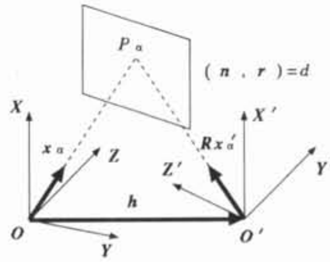

Figure 1: The camera model and the coordinates systems.

Let $\{P_\alpha\}, \alpha = 1, \ldots, N$, be feature points on a planar surface in the scene. Let $\boldsymbol{n}$ be the unit surface normal to the plane, and d the distance of it from the origin O. We call $\{\boldsymbol{n}, d\}$ the *surface parameters* of the plane. As illustrated in Fig. 1, we take the first camera as the reference coordinate system and place the second camera in a position obtained by translating the first camera by vector $\boldsymbol{h}$ and rotating it around the center of the lens by matrix $\boldsymbol{R}$. We call $\{\boldsymbol{R}, \boldsymbol{h}\}$ the *motion* (or *stereo*) *parameters*. The two cameras may have different focal lengths f and f'.

Let $\{(x_\alpha, y_\alpha)\}, \alpha = 1, \ldots, N$, be the image coordinates of the feature points projected on the image plane of the first camera, and $\{(x'_\alpha, y'_\alpha)\}, \alpha = 1, \ldots, N$, those for the second camera. We use the following 3-dimensional vectors to represent them:

$$\boldsymbol{x}_\alpha = \left(\frac{x_\alpha}{f}, \frac{y_\alpha}{f}, 1\right)^\top, \quad \boldsymbol{x}'_\alpha = \left(\frac{x'_\alpha}{f'}, \frac{y'_\alpha}{f'}, 1\right)^\top. \quad (1)$$

In the absence of noise, the vectors $\boldsymbol{x}_\alpha$ and $\boldsymbol{x}'_\alpha$, the motion parameters $\{\boldsymbol{R}, \boldsymbol{h}\}$, and the surface parameters $\{\boldsymbol{n}, d\}$ satisfy the following relation (we omit the derivation [4]):

$$\boldsymbol{x}'_\alpha \times \boldsymbol{A}\boldsymbol{x}_\alpha = \boldsymbol{0}, \quad \boldsymbol{A} = \frac{\boldsymbol{R}^\top(\boldsymbol{h}\boldsymbol{n}^\top - d\boldsymbol{I})}{\sqrt{1+d^2}}. \quad (2)$$

Here, $\boldsymbol{a}\times\boldsymbol{A}$ is the matrix defined by the vector product of 3-dimensional vector $\boldsymbol{a}$ and each column of 3×3-matrix $\boldsymbol{A}$. Let $\boldsymbol{B}_\alpha$ and $\boldsymbol{\nu}$ be a 3×4-matrix and a 4-dimensional vector, respectively, defined by

$$\boldsymbol{B}_\alpha = \left(\boldsymbol{x}'_\alpha \times \boldsymbol{R}^\top\boldsymbol{h}\boldsymbol{x}_\alpha^\top \quad \boldsymbol{x}'_\alpha \times \boldsymbol{R}^\top\boldsymbol{x}_\alpha\right), \quad (3)$$

*E-mail: kanazawa@ice.gunma-ct.ac.jp
†E-mail: kanatani@cs.gunma-u.ac.jp

$$\boldsymbol{\nu} = \frac{1}{\sqrt{1+d^2}} \begin{pmatrix} \boldsymbol{n} \\ -d \end{pmatrix}. \tag{4}$$

Then, Eq. (2) is rewritten in the following form:

$$\boldsymbol{B}_\alpha \boldsymbol{\nu} = \boldsymbol{0}. \tag{5}$$

In the presence of noise, vectors $\boldsymbol{x}_\alpha$ and $\boldsymbol{x}'_\alpha$ do not necessarily satisfy Eq. (5). Write

$$\boldsymbol{x}_\alpha = \bar{\boldsymbol{x}}_\alpha + \Delta \boldsymbol{x}_\alpha, \quad \boldsymbol{x}'_\alpha = \bar{\boldsymbol{x}}'_\alpha + \Delta \boldsymbol{x}'_\alpha, \tag{6}$$

where $\bar{\boldsymbol{x}}_\alpha$ and $\bar{\boldsymbol{x}}'_\alpha$ are the true values of $\boldsymbol{x}_\alpha$ and $\boldsymbol{x}'_\alpha$, respectively. We regard $\Delta \boldsymbol{x}_\alpha$ and $\Delta \boldsymbol{x}'_\alpha$ as random variables that have means 0 and covariance matrices $V[\boldsymbol{x}_\alpha]$ and $V[\boldsymbol{x}'_\alpha]$, respectively [6]. The absolute level of image noise is very difficult to estimate a priori. Let ϵ be the average magnitude of noise, which is unknown. We call it the *noise level.* On the other hand, geometric characteristics of image noise such as uniformity and isotropy can be easily predicted, so we introduce the *normalized covariance matrices* $V_0[\boldsymbol{x}_\alpha]$ and $V_0[\boldsymbol{x}'_\alpha]$, which are assumed to be unknown, and express the covariance matrices in the following form:

$$V[\boldsymbol{x}_\alpha] = \epsilon^2 V_0[\boldsymbol{x}_\alpha], \quad V[\boldsymbol{x}'_\alpha] = \epsilon^2 V_0[\boldsymbol{x}'_\alpha]. \tag{7}$$

3 Optimal Estimation

We apply *maximum likelihood estimation* for estimating an optimal value of $\boldsymbol{\nu}$. First, we optimally correct $\boldsymbol{x}_\alpha$ and $\boldsymbol{x}'_\alpha$ in the form

$$\hat{\boldsymbol{x}}_\alpha = \boldsymbol{x}_\alpha - \Delta \boldsymbol{x}_\alpha, \quad \hat{\boldsymbol{x}}'_\alpha = \boldsymbol{x}'_\alpha - \Delta \boldsymbol{x}'_\alpha, \tag{8}$$

so that Eq. (5) is satisfied for a fixed value of $\boldsymbol{\nu}$. If image noise has a Gaussian distribution, this correction is done for each α by the optimization based on the *Mahalanobis distance* [4] in the form

$$\begin{aligned} J_\alpha &= (\Delta \boldsymbol{x}_\alpha, V_0[\boldsymbol{x}_\alpha]^- \Delta \boldsymbol{x}_\alpha) \\ &\quad + (\Delta \boldsymbol{x}'_\alpha, V_0[\boldsymbol{x}'_\alpha]^- \Delta \boldsymbol{x}'_\alpha) \to \min, \end{aligned} \tag{9}$$

where $V_0[\boldsymbol{x}]^-$ is the generalized inverse of $V_0[\boldsymbol{x}]$ and $(\boldsymbol{a}, \boldsymbol{b})$ denotes the inner product of vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. The residual J_α obtained by substituting the resulting optimal values $\hat{\boldsymbol{x}}_\alpha$ and $\hat{\boldsymbol{x}}'_\alpha$ is a function of $\boldsymbol{\nu}$, so we rewrite it as $J_\alpha[\boldsymbol{\nu}]$ and seek an optimal value of $\boldsymbol{\nu}$ by the minimization

$$\frac{1}{N} \sum_{\alpha=1}^{N} J_\alpha[\boldsymbol{\nu}] \to \min. \tag{10}$$

This minimization is rewritten in the form

$$J[\boldsymbol{\nu}] = \frac{1}{N} \sum_{\alpha=1}^{N} (\boldsymbol{B}_\alpha \boldsymbol{\nu}, \boldsymbol{W}_\alpha(\boldsymbol{\nu}) \boldsymbol{B}_\alpha \boldsymbol{\nu}) \to \min, \tag{11}$$

where

$$\boldsymbol{W}_\alpha(\boldsymbol{\nu}) = \Big(V_0[\boldsymbol{B}_\alpha \boldsymbol{\nu}]\Big)_1^-. \tag{12}$$

The notation $(\cdot)_r^-$ means computing the generalized inverse after projecting the matrix to a matrix of rank r. In Eq.(12), $V_0[\boldsymbol{B}_\alpha \boldsymbol{\nu}]$ is the matrix given in the following form [4]:

$$\begin{aligned} V_0[\boldsymbol{B}_\alpha \boldsymbol{\nu}] &= \bar{\boldsymbol{x}}'_\alpha \times \boldsymbol{A} V_0[\boldsymbol{x}_\alpha] \boldsymbol{A}^\top \times \bar{\boldsymbol{x}}'_\alpha \\ &\quad + (\boldsymbol{A}\bar{\boldsymbol{x}}_\alpha) \times V_0[\boldsymbol{x}'_\alpha] \times (\boldsymbol{A}\bar{\boldsymbol{x}}_\alpha) \\ &\quad + [V_0[\boldsymbol{x}'_\alpha] \times \boldsymbol{A} V_0[\boldsymbol{x}_\alpha] \boldsymbol{A}^\top]. \end{aligned} \tag{13}$$

Here, the vector product $\boldsymbol{A} \times \boldsymbol{a}$ of a 3×3-matrix $\boldsymbol{A}$ and a 3-dimensional vector $\boldsymbol{a}$ is a 3×3-matrix defined by

$$\boldsymbol{A} \times \boldsymbol{a} = (\boldsymbol{a} \times \boldsymbol{A}^\top)^\top. \tag{14}$$

The exterior product $[\boldsymbol{A} \times \boldsymbol{B}]$ of 3×3-matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ is a 3×3-matrix defined by

$$[\boldsymbol{A} \times \boldsymbol{B}]_{ij} = \sum_{k,l,m,n=1}^{3} \epsilon_{ikl} \epsilon_{jmn} A_{km} B_{ln}, \tag{15}$$

where ϵ_{ijk} is the *Eddington's epsilon*, taking values 1, -1, and 0 if (ijk) is obtained from (123) by an even permutation, an odd permutation, and otherwise, respectively.

Let $\boldsymbol{\nu}$ be the optimal solution of the minimization (11) under the constraint $\|\boldsymbol{\nu}\| = 1$. It can be shown that the theoretical covariance matrix of the optimal solution $\boldsymbol{\nu}$ has form

$$V[\boldsymbol{\nu}] = \epsilon^2 \left(\sum_{\alpha=1}^{N} \boldsymbol{P}_{\boldsymbol{\nu}} \boldsymbol{B}_\alpha^\top \boldsymbol{W}_\alpha(\boldsymbol{\nu}) \boldsymbol{B}_\alpha \boldsymbol{P}_{\boldsymbol{\nu}} \right)^-, \tag{16}$$

where $\boldsymbol{P}_{\boldsymbol{\nu}} = \boldsymbol{I} - \boldsymbol{\nu}\boldsymbol{\nu}^\top$ (we omit the proof [4]).

4 Renormalization

If $\boldsymbol{W}_\alpha(\boldsymbol{\nu})$ is replaced by a constant matrix $\boldsymbol{W}_\alpha$, the function $J[\boldsymbol{\nu}]$ in Eq. (11) is written in the following form:

$$J[\boldsymbol{\nu}] = (\boldsymbol{\nu}, \boldsymbol{M}\boldsymbol{\nu}). \tag{17}$$

Here, $\boldsymbol{M}$ is the 4×4-*moment matrix* defined by

$$\boldsymbol{M} = \frac{1}{N} \sum_{\alpha=1}^{N} \boldsymbol{B}_\alpha^\top \boldsymbol{W}_\alpha \boldsymbol{B}_\alpha. \tag{18}$$

The solution that minimizes Eq. (17) under the constraint $\|\boldsymbol{\nu}\| = 1$ is given by the unit eigenvector for the smallest eigenvalue of $\boldsymbol{M}$. It appears at first sight that the optimal solution of Eq. (11) is obtained by letting $\boldsymbol{W}_\alpha = \boldsymbol{W}_\alpha(\boldsymbol{\nu}_0)$ for an appropriate estimate $\boldsymbol{\nu}_0$ and minimizing Eq. (17). Using the resulting solution $\boldsymbol{\nu}_1$, we can update the weight by letting $\boldsymbol{W}_\alpha = \boldsymbol{W}_\alpha(\boldsymbol{\nu}_1)$ and iterate this process until convergence. However, such iterations introduce statistical bias into the solution [3]. This is shown as follows.

Define 4×4-matrices $\boldsymbol{N}^{(1)}$ and $\boldsymbol{N}^{(2)}$ by

$$\boldsymbol{N}^{(1)} = \begin{pmatrix} \frac{1}{N}\sum_{\alpha=1}^{N}\Big((\boldsymbol{h}, \boldsymbol{X}_\alpha \boldsymbol{h}) V_0[\boldsymbol{x}_\alpha] + (\boldsymbol{h}, \boldsymbol{Y}_\alpha \boldsymbol{h}) \boldsymbol{x}_\alpha \boldsymbol{x}_\alpha^\top\Big) & \frac{1}{N}\sum_{\alpha=1}^{N}(V_0[\boldsymbol{x}_\alpha]\boldsymbol{X}_\alpha \boldsymbol{h} + (\boldsymbol{x}_\alpha, \boldsymbol{Y}_\alpha \boldsymbol{h})\boldsymbol{x}_\alpha) \\ \frac{1}{N}\sum_{\alpha=1}^{N}\Big((V_0[\boldsymbol{x}_\alpha]\boldsymbol{X}_\alpha \boldsymbol{h})^\top + (\boldsymbol{x}_\alpha, \boldsymbol{Y}_\alpha \boldsymbol{h})\boldsymbol{x}_\alpha^\top\Big) & \frac{1}{N}\sum_{\alpha=1}^{N}((V_0[\boldsymbol{x}_\alpha]; \boldsymbol{X}_\alpha) + (\boldsymbol{x}_\alpha, \boldsymbol{Y}_\alpha \boldsymbol{x}_\alpha)) \end{pmatrix},$$

$$\boldsymbol{N}^{(2)} = \begin{pmatrix} \frac{1}{N}\sum_{\alpha=1}^{N}(\boldsymbol{h}, \boldsymbol{Y}_\alpha \boldsymbol{h}) V_0[\boldsymbol{x}_\alpha] & \frac{1}{N}\sum_{\alpha=1}^{N} V_0[\boldsymbol{x}_\alpha]\boldsymbol{Y}_\alpha \boldsymbol{h} \\ \frac{1}{N}\sum_{\alpha=1}^{N}(V_0[\boldsymbol{x}_\alpha]\boldsymbol{Y}_\alpha \boldsymbol{h})^\top & \frac{1}{N}\sum_{\alpha=1}^{N}(V_0[\boldsymbol{x}_\alpha]; \boldsymbol{Y}_\alpha) \end{pmatrix}, \tag{19}$$

where $\boldsymbol{X}_\alpha$ and $\boldsymbol{Y}_\alpha$ are 3×3-matrices defined, respectively, by

$$\boldsymbol{X}_\alpha = \boldsymbol{R}(\boldsymbol{x}'_\alpha \times \boldsymbol{W}_\alpha \times \boldsymbol{x}'_\alpha)\boldsymbol{R}^\top, \tag{20}$$

$$\boldsymbol{Y}_\alpha = \boldsymbol{R}[\boldsymbol{W}_\alpha \times V_0[\boldsymbol{x}'_\alpha]]\boldsymbol{R}^\top, \tag{21}$$

and the inner product $(\boldsymbol{A}; \boldsymbol{B})$ of 3×3-matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ is defined by

$$(\boldsymbol{A}; \boldsymbol{B}) = \sum_{i,j=1}^{3} A_{ij} B_{ij}. \tag{22}$$

Define the *unbiased moment matrix* $\hat{\boldsymbol{M}}$ by

$$\hat{\boldsymbol{M}} = \boldsymbol{M} - \epsilon^2 \boldsymbol{N}^{(1)} + \epsilon^4 \boldsymbol{N}^{(2)}. \tag{23}$$

Hence, we can obtain an unbiased estimate of $\boldsymbol{\nu}$ if we use $\hat{\boldsymbol{M}}$ instead of $\boldsymbol{M}$. However, the noise level ϵ is unknown. In order to resolve this difficulty, we introduce a numerical scheme called *renormalization*, which treats ϵ^2 as a variable. The procedure for renormalization is stated as follows [3, 4, 6]:

1. Let $c = 0$ and $\boldsymbol{W}_\alpha = \boldsymbol{I}$, $\alpha = 1, \ldots, N$.
2. Compute the moment matrix $\boldsymbol{M}$ defined by Eq. (18).
3. Compute the 4×4-matrices $\boldsymbol{N}^{(1)}$ and $\boldsymbol{N}^{(2)}$ defined by Eqs. (19), and compute the following 4×4-matrix
$$\hat{\boldsymbol{M}} = \boldsymbol{M} - c\boldsymbol{N}^{(1)} + c^2 \boldsymbol{N}^{(2)}. \tag{24}$$
4. Compute the smallest eigenvalue λ of $\hat{\boldsymbol{M}}$ and the corresponding unit eigenvector $\boldsymbol{\nu}$.
5. If $\lambda \approx 0$, return $\boldsymbol{\nu}$, c and $\hat{\boldsymbol{M}}$. Otherwise, update c and $\boldsymbol{W}_\alpha$ as follows:
$$D = \Big((\boldsymbol{\nu}, \boldsymbol{N}^{(1)}\boldsymbol{\nu}) - 2c(\boldsymbol{\nu}, \boldsymbol{N}^{(2)}\boldsymbol{\nu})\Big)^2 - 4\lambda(\boldsymbol{\nu}, \boldsymbol{N}^{(2)}\boldsymbol{\nu}), \tag{25}$$
$$\text{if } D \geq 0, \quad c \leftarrow c + \frac{(\boldsymbol{\nu}, \boldsymbol{N}^{(1)}\boldsymbol{\nu}) - 2c(\boldsymbol{\nu}, \boldsymbol{N}^{(2)}\boldsymbol{\nu}) - \sqrt{D}}{2(\boldsymbol{\nu}, \boldsymbol{N}^{(2)}\boldsymbol{\nu})},$$
$$\text{if } D < 0, \quad c \leftarrow c + \frac{\lambda}{(\boldsymbol{\nu}, \boldsymbol{N}^{(1)}\boldsymbol{\nu})}, \tag{26}$$
$$\boldsymbol{A} = \boldsymbol{R}^\top(\boldsymbol{h}(\nu_1, \nu_2, \nu_3) + \nu_4 \boldsymbol{I}), \tag{27}$$
$$\boldsymbol{W}_\alpha \leftarrow \Big(\boldsymbol{x}'_\alpha \times \boldsymbol{A}V_0[\boldsymbol{x}_\alpha]\boldsymbol{A}^\top \times \boldsymbol{x}'_\alpha + (\boldsymbol{A}\boldsymbol{x}_\alpha) \times V_0[\boldsymbol{x}_\alpha] \times (\boldsymbol{A}\boldsymbol{x}_\alpha) + c[V_0[\boldsymbol{x}'_\alpha] \times \boldsymbol{A}V_0[\boldsymbol{x}'_\alpha]\boldsymbol{A}^\top]\Big)_1^-. \tag{28}$$
6. Go back to Step 2.

If the vector $\boldsymbol{\nu}$ is obtained, we can compute the surface parameters $\{\boldsymbol{n}, d\}$ of the planar surface in the form form

$$\boldsymbol{n} = N[(\nu_1,\ \nu_2,\ \nu_3)^\top], \quad d = -\frac{\nu_4}{\sqrt{1 - \nu_4^2}}. \tag{29}$$

The symbol $N[\,\cdot\,]$ denotes normalization into a unit vector. An unbiased estimate of the squared noise level ϵ^2 is given in the following form [4]:

$$\hat{\epsilon}^2 = \frac{c}{1 - 3/N}. \tag{30}$$

The covariance matrix $V[\boldsymbol{\nu}]$ given by Eq. (16) is approximated by

$$V[\boldsymbol{\nu}] \approx \frac{\hat{\epsilon}^2}{N}(\hat{\boldsymbol{M}})_3^-. \tag{31}$$

Thus, we can compute by renormalization not only an optimal estimate of $\boldsymbol{\nu}$ but also an estimate of the unknown noise level ϵ and the reliability of the computed estimate $\boldsymbol{\nu}$.

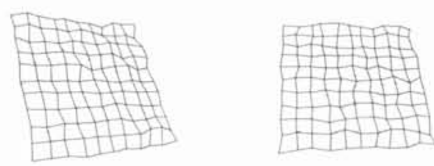

Figure 2: Left and right images with noise.

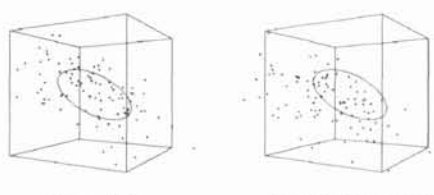

Figure 3: Error vectors.

5 Experiment

Numerical Simulation

We illustrate the effectiveness of our method by doing numerical simulation. We place a grid pattern in a 3-D space and regard the grid points as feature points. The two cameras are assumed to have the same focal length $f = 600$ (pixels). After projecting the feature points onto the image planes, we add as image noise a Gaussian random number with standard deviation 3 (pixels) to each of the image coordinates independently. Hence, the noise level ϵ is equal to $1/200$, and $V_0[\boldsymbol{x}_\alpha] = V_0[\boldsymbol{x}'_\alpha] = \mathrm{diag}(1, 1, 0)$ (the diagonal matrix with 1, 1, 0 as the diagonal elements in that order). However, the value of ϵ is regarded as an unknown in the simulation. Fig. 2 shows the left and right images. The result obtained by our method is shown in Fig. 4. For the sake of comparison, we show the result obtained by the usual least-squares fitting (as described in Section 1) in the same figure. We can observe that our method produces better results than the least-squares method.

Analysis of Error Behavior

We define the *error vector* by

$$\Delta\boldsymbol{u} = \boldsymbol{P}_{\bar{\mathrm{n}}}(\boldsymbol{n} - \bar{\boldsymbol{n}}) + \frac{d - \bar{d}}{\bar{d}}\bar{\boldsymbol{n}}, \tag{32}$$

where we put $\boldsymbol{P}_{\bar{\mathrm{n}}} = \boldsymbol{I} - \bar{\boldsymbol{n}}\bar{\boldsymbol{n}}^\top$ and $\{\bar{\boldsymbol{n}}, \bar{d}\}$ are the true surface parameters. From the theoretical covariance matrix $V[\boldsymbol{\nu}]$ given by Eq. (16), the covariance matrix $V[\boldsymbol{u}]$ of the error vector is computed in the following form (we omit the derivation [4]):

$$V[\boldsymbol{u}] = V[\boldsymbol{n}] + \frac{1}{\bar{d}}(V[\boldsymbol{n}, d]\bar{\boldsymbol{n}}^\top + \bar{\boldsymbol{n}}V[\boldsymbol{n}, d]^\top) + \frac{1}{\bar{d}^2}V[d]\bar{\boldsymbol{n}}\bar{\boldsymbol{n}}^\top. \tag{33}$$

We repeat the computation 100 times, each time using different noise, and plot the error vector three-dimensionally in Fig. 3. The ellipsoids in the figures indicate the theoretical standard deviation in each orientation computed from the covariance matrix given by

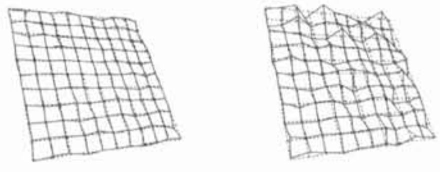

Figure 4: Reconstructed planar surface

Figure 5: Reliability of 3-D reconstruction.

Eq. (16). We can observe that the solution computed by the least-squares method is statistically biased. In contrast, our solution is observed to be statistically unbiased and almost attains the theoretical lower bound.

Reliability of 3-D Reconstruction

The unit eigenvector $\boldsymbol{\xi}_{\max}$ of the covariance matrix $V[\boldsymbol{\nu}]$ for the largest eigenvalue $\lambda_{\max}$ indicates the orientation of the most likely deviation of $\boldsymbol{\nu}$ from its true value, and $\sqrt{\lambda_{\max}}$ indicates the standard deviation in that orientation. Hence, we can visualize the reliability of the reconstructed planar surface by displaying the two planes corresponding to the two vectors

$$\begin{aligned} \boldsymbol{\nu}^{+} &= N[\hat{\boldsymbol{\nu}} + \sqrt{\lambda_{\max}}\boldsymbol{\xi}_{\max}], \\ \boldsymbol{\nu}^{-} &= N[\hat{\boldsymbol{\nu}} - \sqrt{\lambda_{\max}}\boldsymbol{\xi}_{\max}]. \end{aligned} \tag{34}$$

The covariance matrix $V[\boldsymbol{\nu}]$ is computed by the approximation (31) from the data alone. We call these two planes the *primary deviation pair*. The primary deviation pair computed from Fig. 2 is shown in Fig. 5, where the reconstructed grid pattern is drawn in solid lines and the primary deviation pair is drawn in broken lines.

Real-Image Example

Fig. 6 shows two stereo images. The left figure in Fig. 7 shows a grid pattern defined by feature points (corners of the windows) extracted from the left image of Fig. 6. The motion parameters are obtained by the *optimal camera calibration system* [5]. The right

Figure 6: Real stereo images.

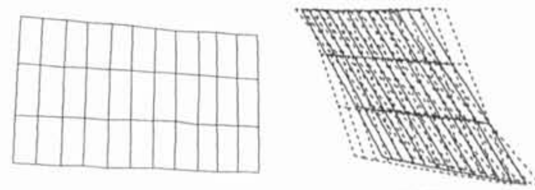

Figure 7: Feature points extracted from the left image and the reliability of its 3-D reconstruction.

figure in Fig. 6 shows the computed 3-D shape. The reconstructed grid pattern is displayed in solid lines, and the primary deviation pair is displayed in broken lines. Thus, we can visualize the reliability of 3-D reconstruction without using any knowledge of the magnitude of image noise. In this experiment, the base line $\|\boldsymbol{h}\|$ is very short as compared with the distance to the building surface (approximately 1/16). Hence, the reliability of 3-D reconstruction is very low. It is very important to evaluate the reliability of 3-D sensing in real applications of stereo for robot operations, because otherwise robots are unable to take appropriate actions to archive given tasks effectively.

6 Conclusion

We have presented a direct reconstruction method for reconstructing a planar surface by stereo vision. By doing numerical simulation, we have shown that the obtained solution almost attains the theoretical lower bound on accuracy. Our method can not only reconstruct the optimal estimate but also allows us to evaluate the reliability of the computed estimate quantitatively. This has a great significance in robot operations in real environments.

References

[1] O. Faugeras: *Three-Dimensional Computer Vision: A Geometric Viewpoint*, MIT Press, Cambridge, MA, U.S.A., 1993.

[2] K. Kanatani: *Geometric Computation for Machine Vision*, Oxford Univ. Press, Oxford, U.K., 1993.

[3] K. Kanatani: "Renormalization for unbiased estimation", *Proc. 4th Int. Conf. Comput. Vision (ICCV'93)*, May, 1993, Berlin, F.R.G., pp. 599–606.

[4] K. Kanatani: *Statistical Optimization for Geometric Computation: Theory and Practice*, Lecture Note, Department of Computer Science, Gunma University, April 1994.

[5] K. Kanatani and T. Maruyama: "Optimal focal length calibration system", *Proc. IEEE/RSJ Int. Conf. Intell. Robots Sys. (IROS'93)*, July 1993, Yokohama, Japan, pp. 1816–1821.

[6] K. Kanatani: "Statistical analysis of geometric computation", *CVGIP: Image Understanding*, Vol. 59, No. 3, pp. 286–306, 1994.

[7] J. Weng, T. S. Huang and N. Ahuja: *Motion and Structure from Image Sequences*, Springer, Berlin, F.R.G., 1993.