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MORPHOLOGICAL ALGORITHMS ADAPTED TO A LINEAR ACQUISITION.

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ABSTRACT

The aim of this paper is to propound an algorithmic approach of the mathematical morphology compatible with a continuous acquisition of the image by a linear camera.In accordance with the three fondamental axis defined by an hexagonal frame , the basic two-dimensional morphological transformations are decomposed into linear processings that can be executed in parallel.This method offers new possibilities concerning iteratives transformations and the use of structuring elements of large size.The application of the method to linear filtering ,issued from the signal theory,is also proposed.

INTRODUCTION

The automation of surfaces inspection (textile industry,treatment of leather,wood,etc...) set up a fundamental axis of search in artificial vision. The human visual control,still necessary today,is a hard work and not always efficient because it depends on the state of tiredness of the operator. To automate such a task ,it is necessary to conceive algorithms which take the implementation on a specialised architure and a real-time processing into account.The huge amounts of information to be processed in an image makes necessary the use of parallelism in differents levels according to the specific criterions of a given application [1] .The suggested algorithms are inspired by the mathematical morphology which is put in concrete form by shape transformations with structuring elements in application of set theory. This method perfectly suited to two-dimensionnal images,provided by an acquisition done with a matrix ,is in difficulty for the inspection systems which often require a linear acquisition.The classical linear morphology implemented line by line may not reconstitute the 2D morphological results because it gives greater importance ,without valid theoretical reason ,to the horizontal direction ;then the horizontal neighbourhood of each pixel is the only one to be taken into account . The object of the paper is to propose a decomposition of the 2D morphological processings into linear and parallel processings [2] .The considered algorithms are the ones that works at pixel level and can be easily implemented on line processors structure [3]. Our interest focuses on a 2D image of a surface in a continuous movement acquired by a linear camera .

IMAGE PARTITION

Let us consider a two-dimensional image I constitued of series of digitased lines,delivered by a linear camera. The hexagonal frame supporting I defines three principal directions;one horizontal noted (D-) and two obliques noted (D/) and (D\) , making respectively an angle of $\pi/3$ and (π - $\pi/3$) with the horizontal (figure 1).The image I can be considered ,independently, as union of lines (Li-) parallel to (D-)or lines (Li/) or (Li\) respectively parallel to (D/) and (D\) :

$$I= U_{i>=1}(Li-) = U_{i>=1}(Li/) = U_{i>=1}(Li\backslash).$$

$$\forall\ i>=1 : (L_i-) // (D-) , (L_i/) // (D/) , (L_i\backslash) // (D\backslash).$$

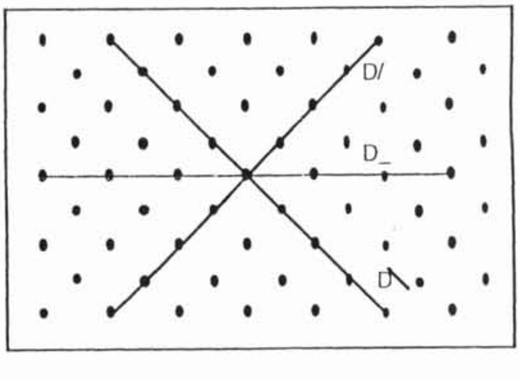


Figure 1.The three fondamental axis defined by the hexagonal frame supporting I.

DECOMPOSITION OF THE BASIC TRANSFORMATIONS

Decomposition of the structuring elements : The mathematical morphology use abundantly the notion of structuring element:elementary configuration of pixels centred on a point which is ,generally,the geometic center and that we move successively over the points of the image ;it will condition the morphological transformations according to its inclusion or intersection with a shape.Let S (Figure 2) be an hexagonal structuring element;its decomposition according to the three principal directions gives:

$$S = (S-) \cup (S/) \cup (S\backslash),$$

with (S-) // (D-), (S/) // (D/) and (S\) // (D\) . Let us note that (S-) , (S/) and (S\) have the same central element ,which is the main actor in the morphological transformations .

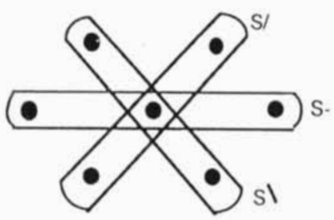


Figure 2 . Decomposition of the structuring element S.

Decomposition of the dilation and the erosion : The two basic transformations of mathematical morphology are dilation and erosion,other transformations such as closing and opening are deduced by simple composition.They are expressed in function of classical set operators of set union and set intersection or in function of addition and substraction (noted respectively $\oplus$ and $\ominus$) operators of Minkowski [4] .

Case of a binary image : The dilation of a shape X by a structuring element S is defined as follows :

$$D^{S}(X) = X\ H{\sim}S^{\sim}=\{\ x,(S)_{x} \cap X \neq \emptyset\}$$

$(S)_x$ designs S centred in x and $S^{\sim}$ the symetrical of S according to the origin of coordinates [4] .Generally S=H ,where H designs the hexagonal structuring element which all pixels have the same level (1 or 0). In that case $H^{\sim} = H$ and $D^{H}(X) =\{x,(H)_x \cap X \neq \emptyset\}$. The decomposition according to the principal axis leads to :

$$D^{H}(X) = D^{H-}(X) \cup D^{H/}(X) \cup D^{H\backslash}(X)$$

$$\text{with } H=(H-)\cup(H/)\cup(H\backslash).$$

Consequently the classical dilation split up into three linear dilations that can be implemented separately. The erosion is the dual operation of dilation ;to erode a shape is the same as to dilate its background :

$$X \ominus H\sim\ = (X^{c} \oplus H\sim)^{c} \text{ which gives:}$$

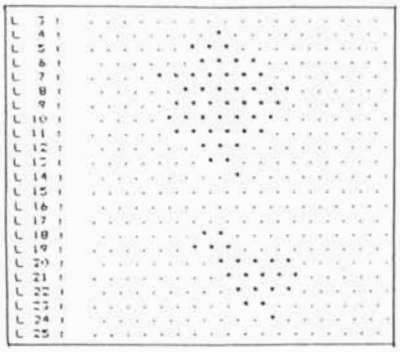
$$E^{H}(X) = x \ominus H{\sim}=\{\ x,(H)_{x} \sqsubset X\}\ .$$

The decomposition of erosion leads to :

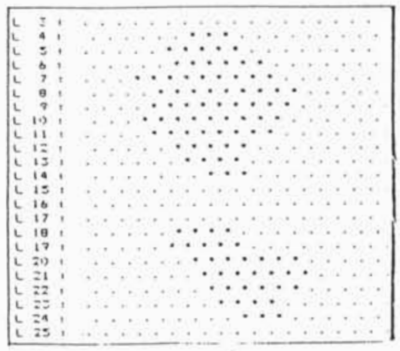
$$E^{H}(X) = E^{H-}(X) \cap E^{H/}(X) \cap E^{H\backslash}(X).$$

To illustrate the proposed method ,here is an example of dilations with an elementary hexagon computed in parallel :

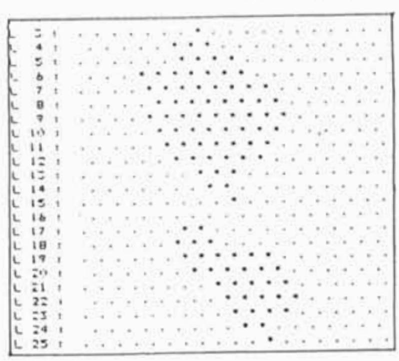
Initial binary image



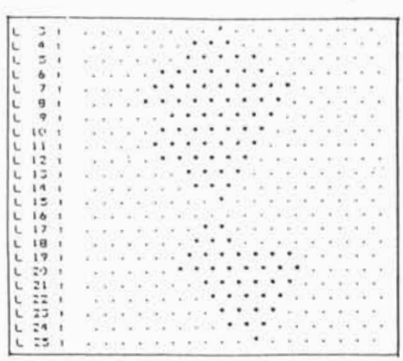
Dilated according to (D-)



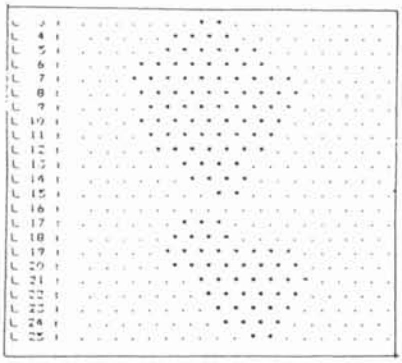
Dilated according to (D/).



Dilated according to (D\).



Synthesis of the parallel dilations.



Case of multilevels image: A grey function n is defined :

$$n : I \quad \text{-------}> [0,N]$$

$$x \quad \text{-------}> n(x)$$

(generally N=256)

The dilation or the erosion transforms the grey level of the pixel in function of levels of its neighborhoods

$$D^S n(x) = \max_{xi \in S} (n(xi)).$$

$$E^S n(x) = \min_{xi \in S} (n(xi)).$$

The linearisation according to D-,D/ and D\ gives :

$$D^S n(x) = \max (D^{S-} n(x), D^{S/} n(x), D^{S\backslash} n(x)).$$

$$E^S n(x) = \min (E^{S-} n(x), E^{S/} n(x), E^{S\backslash} n(x)).$$

EXTENSION TO THE"HIT OR MISS "TRANSFORMATIONS

Let us consider ,in a binary image ,an hexagonal structuring element S(S1,S2); S=S1 U S2 ,

$$S1 = \{x \in S , n(x) = 1\} ; S2 = \{x \in S , n(x)=0\}.$$

The "hit or miss" transformation" of a shape X by a structuring element S(S1,S2) is defined as follows :

$$X \circledast S = \{x ,(S1)x \sqsubset X \text{ and } (S2)x \sqsubset X^c\}.$$

$$S1 = S1\text{-} \cup S1/ \cup S1\backslash \quad ; \quad S2 = S2\text{-} \cup S2/ \cup S2\backslash.$$

$X \circledast S = \{x /(S1\text{-} \cup S1/ \cup S1\backslash)x \sqsubset X$ and $(S2\text{-} \cup S2/ \cup S2\backslash)x \sqsubset X^c\} = \{x/ ((S1\text{-})x \sqsubset X$ and $(S2\text{-})x \sqsubset X^c)$ and $((S1/)x \sqsubset X$ and $(S2/)x \sqsubset X^c)$ and $((S1\backslash)x \sqsubset X$ and $(S2\backslash)x \sqsubset X^c)\}$ so the he linearisation gives :

$$X \circledast S = (X \circledast S\text{-}) \cap (X \circledast S/) \cap (X \circledast S\backslash).$$

The morphological thickening and thinning are defined from the"hit or miss" transformation":

Thickening: The thickening consists of adding to the shape X its "hit or miss" transformed with a structuring element S: $X \odot S = X \cup (X \circledast S)$. It results :

$$X \odot S = (X \odot S\text{-}) \cap (X \odot S/) \cap (X \odot S\backslash).$$

Thinning: The thinning consists of substracting (set substract noted \ )from the shape X its "hit or miss" transformed with a structuring element S:

$$X \bigcirc S = X \backslash (X \circledast S) . \text{ It résults :}$$

$$X \bigcirc S = (X \bigcirc S\text{-}) \cup (X \bigcirc S/) \cup (X \bigcirc S\backslash).$$

EDGES DETECTION

A large class of edges detectors ,after a preprocessing step (usually a sthmoothing) ,computes gradients in different directions for each pixel [5].The morphological gradient can be approximated in function of grey level erosion and dilation [6]by :

$$g(x) = (D^S n(x) - E^S n(x))/2 .$$

Consequently ,the gradient g can be decomposed according to the three principal axis and leads to a parallel processing in order to extract edges with an appropriate threshholding.

In the particularly case of a binary image ,we consider a neighbourhood family V of structuring elements engendered by the structuring element V1 which center has the level 1 and which has at least one pixel at the level 0;the other points have indiscriminately the value 1 or 0. V can be reduced to 6 elements V1,V2 ... V6 ,each Vi is deduced from V_{i-1} by a rotation of a $\pi/3$ angle. The contour C(X) of a shape X can be considered as the union :
$(X \circledast V_i)\ i \in \{1,2,...,6\} = \cup_{i \in \{1,..,6\}} (X \circledast V_i)$ [7]. The family V can be decomposed according to the principal axis :
$V = (V_{1\text{-}} \cup V_{2\text{-}}) \cup (V_{1/} \cup V_{2/}) \cup (V_{1\backslash} \cup V_{2\backslash})$. It results:
$C(X) = \cup_{i \in [1,2]} ((X \circledast V_{i\text{-}}) \cup (X \circledast V_{i/}) \cup (X \circledast V_{i\backslash}))$.

Here again ,the transformations according to the fondamental directions can be processed in parallel.

ITERATIVES TRANSFORMATIONS

In the classical 2D morphological transforformations, we are often lead to repeat several times basic transformations such as erosion and dilation . In the case of the image of a surface in a continuous movement , the iteration can hardly be enviseaged.The iteration of a morphological transformation using a structuring element S is equivalent to using a structuring element of a larger size but the linear existing systems ,based on the processing of three successives lines find their limit from the moment that the structuring element stragger over more than three lines [8] ;in fact let us consider the binary dilation : the operator $\oplus$ is associative so :

$$(X \oplus H) \oplus H = X \oplus (H \oplus H).$$

$H \oplus H$ is an hexagon which contains 19 pixels and spread over five successives lines. More generally , if i is the number of iteration of $X \oplus H$ it is equal to use the hexagon :

$$H_i = (H \oplus H \oplus H \oplus \dots \oplus H)$$

(where H is repeated i times). If Card(Hi) designs the number of pixels of Hi ,we have :
Card (H_i) = Card (H_{i}-1) + 6i , then it results : card(Hi) = 1 +3i.(i + 1) .Hi have (1 + 3i(i + 1)) elements spreaded over (2i + 1) lines.We note that the number of elements of Hi increases in i^2 ; using such structuring elements ,in a classical way,became a very fast complicated. The decomposition of H according to the three fondamental axis offers an easier solution using linear structuring elements ,each one containing (2i +1) pixels (in function of Hi) :If H = H- U H / U H\, we can verify (Figure 3):

$$H \oplus H = H- \oplus H/ \oplus H\backslash ;$$

$$X \oplus Hi = ((X \oplus (H- \oplus H/ \oplus H\backslash) \oplus \dots \oplus (H- \oplus H/ \oplus H\backslash)).$$

Each couple of the set {H- , H/ , H\ } commutes by $\oplus$.
if i=2k ,Hi =Hk- $\oplus$ Hk/ $\oplus$ Hk\ and by successives commutations and by the fact that $\oplus$ is associative we obtain :

$$X \oplus Hi = (((X \oplus Hk-) \oplus Hk/) \oplus Hk\backslash)$$

H- H- $\oplus$ H/ H- $\oplus$ H/ $\oplus$ H\
= H $\oplus$ H

Figure 3.Action of linear dilations.

The duality of erosion and dilation leads ,in the same way to:

$$X \ominus Hi = (((X \ominus Hk-) \ominus Hk/) \ominus Hk\backslash).$$

This result suggests a flow parallelism by the fact that each pixel is subject to a spatiotemporal processing at different moments corresponding to the three fondamental directions.

APPLICATION TO THE LINEAR FILTERING

The linear filters derive from the signal processing; they are essentially convolution products.Let n be a grey function and V(x) a neighbourhood centred in the point x_0 which is treated .The convolution product defines an image function g(x) by:

$$g(x) = f(x) * V(x)$$

If i designs the number of neighbours of x_0 noted x_i the convolution product gives :

$$g(x)= \sum_{i=0}^{n} f(x_i) * V(x_i)$$

The considered neighbourhood which can be hexagonal act as a structuring element and consequently may be divided in accordance with the directions D-,D/ and D\ :

$$V(x) = V\text{-}(x) \cup V/(x) \cup V\backslash(x)$$

The convolution product became :

$$g(x) = f(x_0).V(x_0) + g\text{-}(x) + g/(x) + g\backslash(x)$$

With :

$$g\text{-}(x)= \sum_{x_i \in V\text{-}(x)} f(x_i) * V\text{-}(x_i)$$

$$g/(x)= \sum_{x_i \in V/(x)} f(x_i) * V/(x_i)$$

$$g\backslash(x)= \sum_{x_i \in V\backslash(x)} f(x_i) * V\backslash(x_i)$$

Let us note that the convolution product is generally normalized ; it is divided by the sum of the coefficints of the neighbourhood function :

$$g'(x) = g(x) / (\sum_{i=0}^{n} V(x_i))$$

The neighbourhood may enclose other pixels than the nearest neighbours of the pixel x_0 but the use of a large size neighbourhood is generally avoided because it needs numerous operations or several iterations of a filtering using a small neighbourhood. The use of linear neighbourhoods ,actued by our method is more advisable when a real-time processing is envisaged.

CONCLUSION

The exposed algorithms allow to change from the 2D morphology to a linear morphology which is more adapted to an acquisition line by line of the image.The hexagonal frame has been choosen as support of image because it is the most acceptable theoretically byt the fact of its isotropy.However all the obtained results can be transposed in the case of a square frame . This method permit to effect at one stretch transformations that are classically iteratives and may be extented to the linear filters which act on the image in the same way as the structuring elements . It leads to a parallelisation of processings and authorizes an implementation on an architecture essentially built on line processors.

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