

## IDENTIFICATION OF CORE AND DELTA POINTS IN FINGERPRINT IMAGES

V.S.Srinivasan

CMC Ltd, 115, Sarojini Devi Road, SECUNDERABAD - 500 003 ( INDIA )

---

### Abstract

A technique for the machine detection of the core and delta points in a fingerprint image is presented here. The core and delta points are used as points of registration in fingerprint matching. The method makes use of the structural information derived from the histograms of the directional image of the fingerprint. The directional image is obtained by taking the dominant direction in each sub-block of the image. To detect the coordinates of the points in the image space, an intelligent tracking is carried out using the thinned image. The usage of the histograms allows for a high degree of noise tolerance.

---

### Introduction

Any automatic fingerprint identification system's effectiveness is determined from the accuracy with which the system matches the search prints with the file prints. To perform matching, certain ridge characteristics, known as minutiae, are used. The number of these minutiae is typically around 100. The matching algorithms are based on point pattern matching (Qin-ghan<sup>(1)</sup>, CMC<sup>(2)</sup>) as compared to an image to image match. The methods of point pattern matching become easier if the points of registration are known. These points, in forensic parlance, are called the core and delta points.

To aid in archiving as well as to cut down the search in the database, fingerprints have been classified into five major classes, namely the loops- left, right and double, whorls and arches (Fig. 1 ). In all these pattern classes, except the arches, the core and delta points have been defined. Nevertheless, in prints from the scene of crime, where only partial prints are available, it would still be necessary to have algorithms that match without the points of registration being available.

Methods for the automatic detection of these points using syntactic tree grammars have been reported by Rao<sup>(3,4,5)</sup>. Kawagoe<sup>(6)</sup> uses an approach which is a mix of a relaxation method (for noise reduction) and a pruning based on the neighbours. These methods rely on the accuracy of the immediate neighbourhood information. In real life cases where prints could be noisy ( due to smudging, over-inking, excessive pressure while fingerprinting ), such methods would cause local minima to be detected (noisy points locally having the same characteristics as that of the singular points ).

The approach proposed here makes use of the directional histograms in neighbourhoods around these points, thus averaging the effect of any noise points present.

---

### Notations and Definitions

In the following sections, the gray level image of a fingerprint is referred to by G ( of size g x g ). Operations are performed on

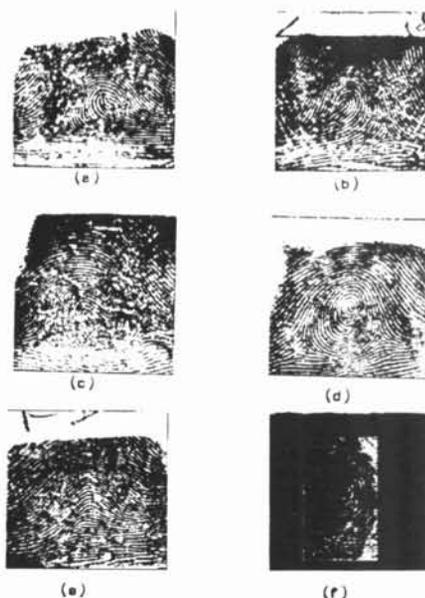


Fig. 1. Some fingerprint pattern classes  
(a)Left loop (b)Right loop (c)Double loop (d)Whorls (e)Arches (f)Print from the scene of crime

the directional histogram formed from the directional image D. The direction at each pixel of D represents the local orientation of the ridge at that pixel. The direction  $D(i,j)$  at pixel  $(i,j)$  in an image { Mehre<sup>(10,11)</sup> } is the direction  $d$  for which  $S_d$ , the sum of differences in gray values along a direction  $d$  is a minimum. The sum of differences  $S_d$  is defined as

$$S_d = \sum_{m=1}^M | G(i_m, j_m) - G(i, j) | \text{ for } d = 1, N$$

where  $G(i,j)$  and  $G(i_m, j_m)$  are the gray values at the pixels  $(i,j)$  and  $(i_m, j_m)$ . The pixel corresponding to  $(i_m, j_m)$  is the  $m^{\text{th}}$  pixel along the direction  $d$  from  $(i,j)$ ,  $M$  being the number of pixels chosen for this computation and  $N$  the number of directions used (direction  $d$  corresponds to an angle of  $\pi/N * [d-1]$  radians). A block directional image B (of size  $b \times b$ ) is obtained by finding the dominant direction in each block (  $g/b \times g/b$  ) of the directional image D. In our case, we have used  $N=8$ ,  $b=16$  and  $g=512$ .

The core and delta points can be defined for all fingerprint pattern classes except arches, more specifically, a single core point and one or two delta points. The fingerprint images can be essentially considered to be two valued images made of ridges and valleys. Over and above the binary nature of the fingerprints, these images can also be characterised by the direction of the ridge flow.

The registration point is termed a

[a] Core point if it is the topmost point on the innermost recurring ridge.

[b] Delta point if it is a point in a delta-like region lying on ridges which extend to encompass the complete pattern area.

The directional histogram is computed from the frequency of each direction in a neighbourhood (nxn). The histogram H is an 8-tuple { H(0) ... H(7) } where H(j) is the frequency of direction j in the neighbourhood defined. W is a set of weights {w0 ... w7} where wj is an 8-tuple {wj(0) ... wj(7)} such that wj(i) is maximum when j = i.

A directional histogram is said to be

[a] Left skewed if directions less than or equal to 90 degrees are predominant.

[b] Right skewed if directions greater than or equal to 90 degrees are predominant.

[c] Edge skewed if the directions not in the range of 45 degrees to 135 degrees are predominant.

A core point then by definition should have the histogram, of it's left neighbourhood left skewed and the right , right skewed. Further for a delta point, the histogram of the bottom neighbourhood should be edge skewed.

#### Detection of core and delta points

Since the block directional image B exhibits the same characteristics as that of the original high resolution image, the singular points can be detected to block accuracy from the same.

#### Selection

The initial set of probable candidates for the core and delta points are determined using :

[i] Core points CP : All points exhibiting a directional difference greater than k1 ( k1 = 2 i.e. 45 degrees) in the immediate neighbourhood. In other words  $d \{ B(i,j) - B(i-1,j) \} > k_1$  where d is the modulo 8 operator.

[ii] Delta points DP : Points with neighbourhoods of the form

$$\begin{array}{|c|c|c|} \hline \times & B(i,j) & \times \\ \hline B(i-1,j+1) & \times & B(i+1,j+1) \\ \hline \end{array}$$

where  $B(i-1,j+1) \leq B(i,j) < B(i+1,j+1)$  or  $B(i-1,j+1) < B(i,j) \leq B(i+1,j+1)$ ,  $B(i-1,j+1) < k_2$  and  $B(i+1,j+1) > k_2$  (k2 = 4 i.e. 90 degrees since we have used N = 8 ).

#### Pruning

For each of the delta points DPk(i,j), for an n21xn21 (n21=5) neighbourhood around the point, form directional histograms H21, H22 and H23 of regions R1,R2 and R3 (Fig. 2 ) respectively. By definition, the histograms of region R1 should be left skewed, R2 should be edge skewed and R3 should be right skewed.

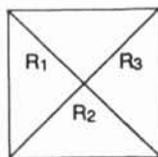


Fig 2. Neighbourhood for deltas

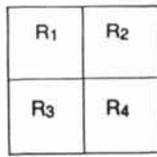


Fig 3. Neighbourhood for cores

Mark for deletion all points not satisfying the following

criteria.

$$P = \sum_{k=0}^7 H_{21}(k) > k_3, P > \sum_{k=0}^7 H_{21}(k) \text{ and } P > \sum_{k=0}^2 H_{21}(k) + \sum_{k=4}^7 H_{21}(k)$$

Also for each of the core points CPk(i,j), for an n22xn22 ( n22 =5) neighbourhood around the point, form a histogram H24. This histogram should have the following property by definition, i.e. it should not have any predominant direction.

$$\text{Max } f_2 ( H_{24}(k) ) < k_4 \text{ where } f_2 ( H(k) ) = \sum_{j=(k-1) \bmod 8}^{(k+1) \bmod 8} H(j)$$

Also, mark for deletion all core points in the n23xn23 (n23=3) neighbourhood of the delta points DP.

#### Curvature typing

Core points typically exhibit a curvature which is either convex ( loops - left, right and whorls) or concave (double loops). Each of the core points CP detected so far is classified into one of these types by forming histograms H31, H32, H33 and H34 of regions R1, R2, R3 and R4 (Fig. 3 ) respectively (in a neighbourhood of n3xn3)

A point with convex curvature should have H31 left skewed and H32 right skewed and those with concave curvature should have H33 right skewed and H34 left skewed.

$$\text{Curv} ( CP_k(i,j) ) =$$

convex if  $f_3 ( H_{31}, H_{32} ) > f_3 ( H_{33}, H_{34} )$  and  $f_3 ( H_{31}, H_{32} ) > k_5$   
 concave if  $f_3 ( H_{31}, H_{32} ) < f_3 ( H_{33}, H_{34} )$  and  $f_3 ( H_{33}, H_{34} ) > k_5$

where  $f_3 ( H_{3m}, H_{3n} ) = \sum_{j=0}^7 w_{\alpha}(j) H_{3m}(j) + \sum_{j=0}^7 w_{\beta}(j) H_{3n}(j)$ ,  $\alpha$  and  $\beta$  being the required orientations for the corresponding regions.

If a core point does not exhibit a clearly defined curvature, that point is marked for deletion.

#### Clustering

All the core and delta points detected so far are clustered (with a clustering algorithm such as k-means, for instance) by using a weighted distance measure giving neighbourhood boxes BCPj = 1,C and BDP j , j = 1,D where C and D are the number of core and delta clusters respectively. BCP and BDP are 4-tuples of the form (x,y,dx,dy). Further search for the core and delta points can be restricted to these boxes.

#### Finding the axis of the pattern area

Once the clusters are determined, for each of the clusters of the core points an axis Ak can be defined. This axis has a direct correspondence with the orientation of the fingerprint pattern. The axis  $\alpha$  of Ak is determined from the histograms H5  $\alpha, \alpha=0,7$  of neighbourhoods n5 $\alpha$ xm5 $\alpha$  with orientation  $\alpha$  around the cluster centre (Fig. 4 ).

$$f_{5\alpha}(H_{5\alpha}) > f_{5\mu}(H_{5\mu}) \forall \alpha < \mu, 0 \leq \alpha \leq 7, 0 \leq \mu \leq 7 \text{ and}$$

$$f_{5\mu}(H) = \sum_{i=0}^7 w_{\mu}(i) H(i), w_{\mu} \text{ being the set of weights for orientation } \mu.$$

#### Core and Delta points

For all points in each of BCP and BDP respectively, points having the maximum probability P of being the singular points are determined.

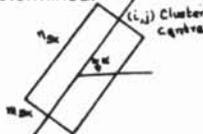


Fig. 4. Neighbourhood for axis



Fig. 5. Neighbourhood for axis of  $\alpha$

For the delta points, the probability  $P$  is a function of the histograms  $H_{61}$ ,  $H_{62}$  and  $H_{63}$  of regions  $R_1$ ,  $R_2$  and  $R_3$  (Fig. 2) respectively.

$$P = \sum_{\beta=1}^7 f_{6\beta} ( H_{6\beta} )$$
, where  $f_{6\beta} ( H ) = \sum_{i=0}^7 w_{\beta} (i) H(i)$ ,  $\beta$  being the orientation for its corresponding region and  $w_{\beta}$  the set of weights for orientation  $\beta$ .

Similarly for core points, this probability is a function of histograms  $H_{71}$ ,  $H_{72}$ ,  $H_{73}$  and  $H_{74}$  of regions  $R_1$ ,  $R_2$ ,  $R_3$  and  $R_4$  respectively for axis A (Fig. 5) for those points which have convex curvatures.

$$P = \sum_{\beta=1}^4 f_{7\beta} H_{7\beta} \text{ where } f_{7\beta} ( H ) = \sum_{i=0}^7 w_{\beta} (i) H(i)$$

The reason for not considering the points with concave curvature are that by definition the core points are the topmost points on the innermost recurring ridge and the core points ought to lie on ridges which are curving downwards.

#### Core and delta points in Gray level image space

For each of the core points  $C(bx_i, by_i)$  detected from the block directional image B, a search in the image space in an area  $( cx_i, cy_i, cdx_i, cdy_i )$  is carried out, where  $cx_i = (bx_i - 3) * (g/b)$ ,  $cy_i = (by_i - 3) * (g/b)$  and  $cdx_i, cdy_i$  are the search area widths.

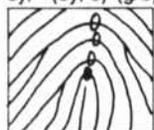


Fig. 6. The region around the core point.

The gray image is first thresholded in this area and then thinned (Pavlidis<sup>(9)</sup>). On this thinned image, each of the ridges are traced to determine the highestmost point. The ridge should have a minimum length MINLEN on both sides of the highestmost point (Fig 6). The ridge should also exhibit a marked change in direction ( by definition, the ridge enclosing the core point curves upon itself). The lowermost of such

points detected is taken as the core point.

While tracking for a core point, when a bifurcation point is encountered, the ridges are followed on both branches and the longer branch is considered, in cases where the ridge being followed is the root branch of the bifurcation. If the ridge being followed happens to be one of the branches of the bifurcation, then the ridge is traced in the direction of the root. The method outlined here could also reject some areas if the areas do not exhibit the required characteristics.

Algorithm : Search for core point

- [i] Initialise the core point to the top left corner of the search area.
- [ii] Scan the thinned image TOP to BOTTOM, LEFT to RIGHT
- [iii] for each non-zero pixel do {
- [iv] initialize the highest point on the ridge as the first non-zero pixel's coordinate
- [v] track the points to the RIGHT, determining the highest point and the length of the left and right branches from the highestmost point
- [vi] similarly, track the points to the LEFT
- [vii] if the lengths of the branches exceed a minimum distance MINLEN and the highestmost point is lower than the core point, then update the core point.}

For a delta point, the same procedure as was described is repeated over a sub block directional image. The sub block directional image is obtained by finding the characteristic direction of the ridges in smaller blocks than those of the block directional image. This was necessitated by the fact that often the area around the delta is smudged and only the ridge flow can be inferred from the ridge. The smaller block size is chosen such that the delta point obtained from it is of the desired accuracy.

## Results

The technique illustrated here has been found to work successfully over a large number of test prints and the accuracy of the points detected was of a high degree. After the points were detected roughly in the block directional image, a further tracking was carried out in the original gray image G to arrive at the  $(x,y)$  coordinates of the points. Fig 8 shows a double loop pattern ( which has points of both curvatures and two delta points) and its block directional image B. Fig 9 and 10 show the core points after pruning and curvature typing. Fig 11 shows the probable core and delta points after step 3.

The axis of the pattern area around the core points of both types and the search areas are shown in Fig 12 and 13. The core and delta points finally detected are marked in Fig 14 which correspond very well to those that a fingerprint expert would identify.

## Conclusion

The approach described here presents a technique to automatically determine the core and delta points of a fingerprint image. The usage of the histogram to determine the structural features such as the core and delta points allows for a fairly high degree of noise tolerance. A procedure for the finer tracking of the core and delta points in the areas detected from the block directional image is also presented here. The method described here has been found to work very well when tested over a large number of fingerprint images. Further, the method provides interesting cues for automatic pattern classification.

## References

- [1] Qinghan Xiao et al , An approach to fingerprint identification by using the attributes of feature lines of fingerprints, Proc Eighth Intl conf on Pattern Recognition, Paris, Oct,1986.
- [2] Communications from the FACTS Matcher Group, CMC Ltd, Secunderabad, 1989-90.
- [3] C.V.Kameshwara Rao et al, Finding the core point in a fingerprint, IEEE Trans on Computers, Jan 1978.
- [4] C.V.Kameshwara Rao et al, Type identification of fingerprints - a syntactic approach, Proc Third Intl conf on Pattern Recognition, San Deigo, Nov, 1976.
- [5] C.V.Kameshwara Rao et al, Automatic fingerprint classification system, Proc Second Intl conf on Pattern Recognition, Copenhagen, 1974.
- [6] M.Kawagoe et al, Fingerprint Pattern Classification, Pattern Recognition, Vol 17 Number3, 1984.
- [7] B.M.Mehtre et al , Segmentation of fingerprint images using the directional image, Pattern Recognition, Vol 20 Number 4, 1987.
- [8] B.M.Mehtre et al , Segmentation of fingerprint images - a composite method, Pattern Recognition, Vol 22 Number 4, 1989.
- [9] Pavlidis.Theo., Algorithms for Graphics and Image Processing , Computer Science Press, 1981.

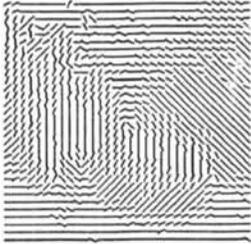


Fig. 8. Block directional image B of a double-loop pattern



Fig. 9. The probable core points after pruning

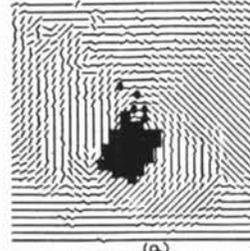


Fig. 12. The axis of pattern area for points of type (a) convex curvature (b) concave curvature

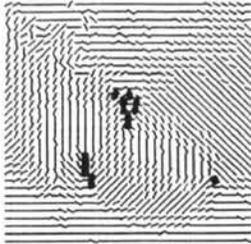


Fig. 10. The core points after curvature typing  
●: cores ■: deltas

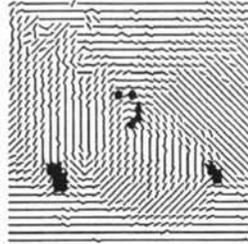


Fig. 11. The core and delta points after step 3. ●: cores ■: deltas

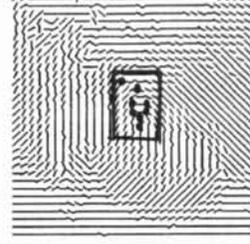


Fig. 13(a) The search area for the core points

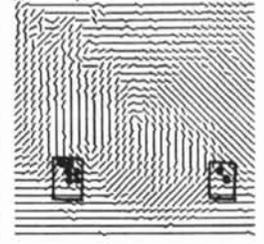


Fig. 13(b) The search area for the delta points

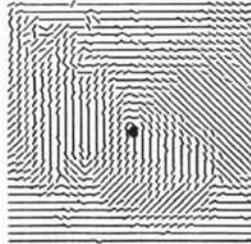


Fig. 14(a). The set of core points

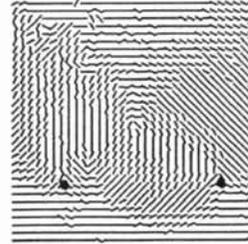


Fig. 14(b). The set of delta points

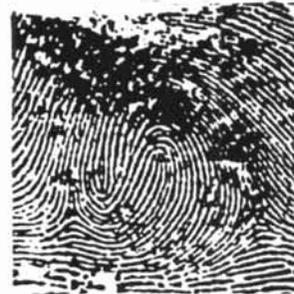


Fig. 7. Image of a double loop pattern

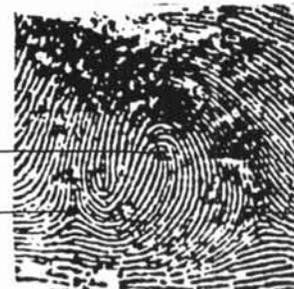


Fig. 15. The core and delta points detected on the image of a double loop pattern  
CORE ← LEFT DELTA → RIGHT DELTA