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SHAPE AND STRAIN MEASUREMENT OF 3-D OBJECT USING FOURIER TRANSFORM GRID METHOD

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ABSTRACT

A grid method for measuring the shape and strain distribution of 3D objects by using the Fourier transform grid method is presented. In the conventional automated grid method, the position of a line is expressed in an integer, so it is difficult to analyze the accurate position. Using this two-dimensional Fourier transform grid method the fault can be overcome by analyzing and interpolating the phase distribution. Using it, the two dimensional grating can be easily separated to x- and y-grating based on its orthogonality.

1 INTRODUCTION

A variety of automated grid methods have been used in the attempt to achieve high-precision measurement. For measuring the strain and shape on the full surface of an object, we have developed an automated grid measurement system using stereoscopic method[1].

However, the position of the grating line is measured with an integer pixel length in the digital image, so it is difficult to measure accurately. For interpolating data between lines, Sciammarella[2] has presented a method for analyzing the phases of mismatched fringes by using the one-dimensional Fourier Transform. Moreover, Takeda and Mutoh[3] have developed the Fourier transform profilometry(FTP). On this method, the shape of an object can be measured by calculating the phases of the Fourier spectrum of the grating projected on the object. On the other hand, we have proposed a new moire method[4] using the first harmonic of the Fourier spectra of the deformed grating to analyze the strain distribution on a plane surface. We call this method the Fourier Transform Moire and Grid Method(FTMGM).

By combining the above methods and the image fusion technique, we subsequently present two Fourier transform grid methods in this paper. One is the shape measurement for cylindrical shape, by analyzing a projected line image. The other is the measurement of shape and strain on the full surface of a three-dimensional object by analyzing the line image recorded from two different directions. Using the Fourier transform method, we not only find the accurate positions of the grid lines, but also identify the corresponding points in the different images which are recorded from different directions by interpolating . So it is possible to measure the three-dimensional shape and strain on full surface of an object by analyzing the images from two directions.

2 PRINCIPLE OF MEASUREMENT

2.1 Shape Measurement

In order to perform noncontact measurement of shape, an automated measurement system illustrated in Fig.1 is developed. A vertical thin beam light emitted from a laser is projected onto the object. The deformed thin light on the surface of the object is recorded by a CCD camera and digitized on an image processor. Figure 2 shows the geometrical relation of the system. The radial

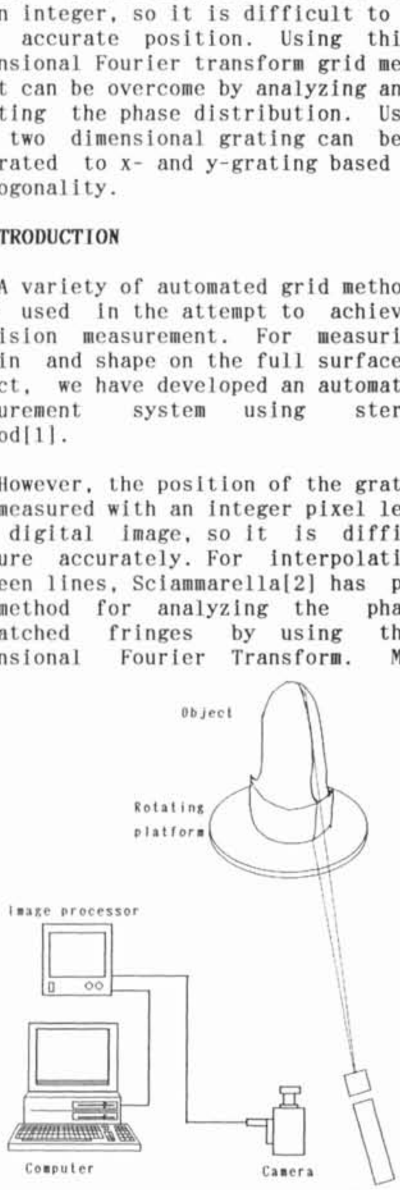

Fig. 1 Measurement system

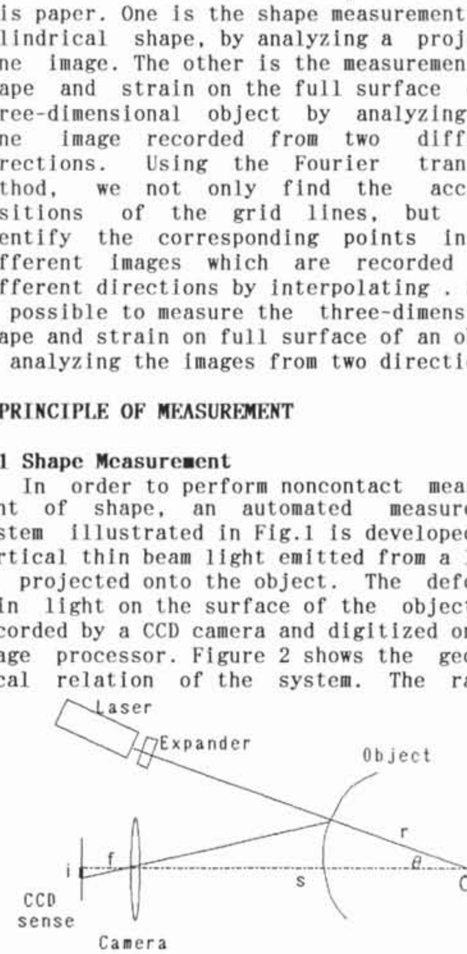

Fig. 2 Geometrical relation of system

distance r of a point on the object can be calculated from the displacement of the corresponding point on the video image using Eq.(1).

$$r = \frac{i \cdot s}{f \cdot \sin\theta + i \cdot \cos\theta} \qquad \text{--------(1)}$$

For measuring the full surface of the object, the image is shifted a constant pixels after rotating the platform a constant angle, and this operation is repeated until the full surface of the object is recorded. Thus the resultant image shows a grating pattern, and it can be analyzed by the Fourier transform grid method. The profile of the object can be described by the angle and its corresponding radial distance.

2.2 Shape and Strain Measurement

In order to measure the strain of an object, two-dimensional grid lines are drawn for identifying the corresponding points before and after deformation. The object is put on a rotatable platform shown in Figure 3. These grid lines also can be used as the corresponding points of the two images recorded from the different directions in the stereoscopic method. If each point on the object is input to the images two times by rotating the platform, the profile of the object is analyzed using the geometrical relation of these corresponding points or following equations.

$$\frac{s+\ell \cdot \sin(\theta+\alpha_1)}{\ell \cdot \cos(\theta+\alpha_1)} = \frac{f}{a} \qquad \text{--------(2)}$$

$$\frac{s+\ell \cdot \sin(\theta+\alpha_2)}{\ell \cdot \cos(\theta+\alpha_2)} = \frac{f}{b} \qquad \text{--------(3)}$$

$$\frac{s+\ell \cdot \sin(\theta+\alpha_1)}{y} = \frac{f}{c} \qquad \text{--------(4)}$$

$$x = \frac{f \cdot s \cdot [(a-f) \cdot \cos\alpha_1 + (f-b) \cdot \cos\alpha_2]}{(a \cdot b - f^2) \cdot \sin(\alpha_1 - \alpha_2) + f \cdot (b-a) \cdot \sin(\alpha_1 + \alpha_2)} \qquad \text{-------------(5)}$$

$$z = \frac{f \cdot s \cdot [-(a+f) \cdot \sin\alpha_1 + (f+b) \cdot \sin\alpha_2]}{(a \cdot b - f^2) \cdot \sin(\alpha_1 - \alpha_2) + f \cdot (b-a) \cdot \sin(\alpha_1 + \alpha_2)} \qquad \text{-------------(6)}$$

$$y = \frac{c \cdot [s + z \cdot \cos\alpha_1 + x \cdot \sin\alpha_1]}{f} \qquad \text{-------(7)}$$

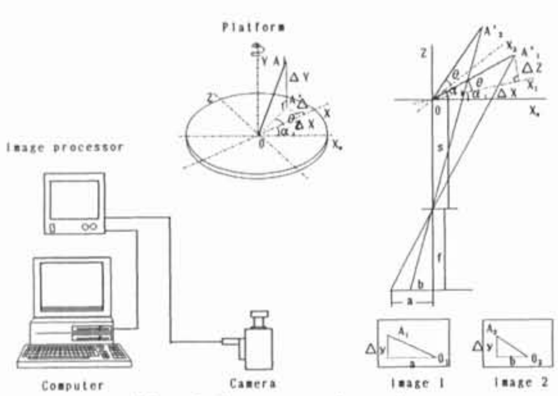

Fig. 3 Measurement system

Using this measurement method, each cross point of the grating on the object can be described in a three-dimensional domain both before and after the deformation. So the strain of the object can be easily calculated as follows:

$$\varepsilon_x = \frac{du}{dx} \qquad \text{----------------(8)}$$

$$\varepsilon_y = \frac{dv}{dy} \qquad \text{----------------(9)}$$

$$\gamma_{xy} = \frac{du}{dx} + \frac{dv}{dy} \qquad \text{---------------(10)}$$

u and v are the components of displacement in the x and y directions respectively. ε_x , ε_y and γ_{xy} are x- and y-directional normal and shear strain[5].

3 THEORY OF FOURIER TRANSFORM METHOD

In the conventional automated grid methods, the space between two grid lines is measured as an integer pixel length. Therefore it is difficult to obtain the accurate positions of the grid lines by directly scanning tracing method[1]. The FTMGM[6] is introduced here for obtain the position in decimal pixel by interpolating.

In order to analyze a two-dimensional grid image, a cross grating shown in Figure 4 is employed. The brightness intensity function of the cross grating can be expressed as the product of two single grating intensity functions. The single grating whose lines are normal to the x axis is called as x-grating, the other one which is normal to the y axis is called as y-grating.

The expansion of the intensity function of the cross grating f(x,y) in Fourier series is

$$f(x,y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} C_{m,n} \exp[j2\pi(m\omega_x x + n\omega_y y)] \qquad \text{------(11)}$$

Where $C_{m,n}$ is the coefficient of the harmonic of the order (m,n), m and n are integers, j is the imaginary unit, and ω_x and ω_y are the frequencies of the gratings defined as

$$\omega_x = 1/p_x, \qquad \omega_y = 1/p_y \qquad \text{-----(12)}$$

p_x and p_y are the pitches of x- and y-gratings respectively.

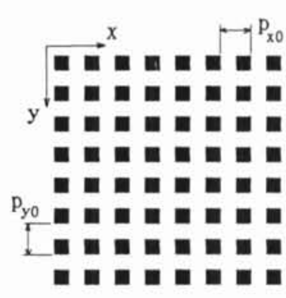

Fig.4 Two-dimensional grating

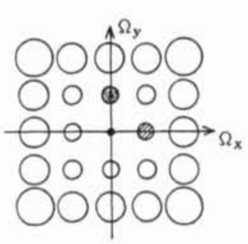

Fig.5 Fourier spectra of Fig.4

By calculating the two-dimensional Fourier transform [DFT] of the grating pattern Eq. (11), we obtain

$$g(\Omega x, \Omega y) = \iint f(x, y) \exp[j2\pi(\Omega_x x + \Omega_y y)] dx dy = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} I_{m,n}(\Omega_x, \Omega_y) \quad ---(13)$$

where $I_{m,n}(\Omega_x, \Omega_y)$ is the Fourier transform of $C_{m,n}$. Ω_x and Ω_y are the components of the frequency vector. Figure 5 shows schematically the spectra obtained by the Fourier transform. Each circle shows the region in which the spectrum of the order (m,n) of the harmonics exists.

According to the orthogonal characteristic, we can extract the first harmonic in X- and Y-direction respectively so that the one-dimensional analysis method becomes adaptable. Let us consider the case in X-direction, we extract the (1,0) order harmonic indicated with oblique lines in Fig.5 by filtering. By computing the inverse Fourier transform [IDFT] of this harmonic $I_{1,0}(x,y)$, we obtain

$$i_{1,0}(x, y) = \iint C_{1,0}(\Omega_x, \Omega_y) \exp[-j2\pi(\Omega_x x - \Omega_y y)] d\Omega_x d\Omega_y = C_{1,0} \exp(j\theta_x(x, y)) \quad -------(14)$$

The real and imaginary parts of Eq.(14) are

$$Re(i_{1,0}(x, y)) = C_{1,0} \cos(\theta_x(x, y)) \quad ----(15)$$

$$Im(i_{1,0}(x, y)) = C_{1,0} \sin(\theta_x(x, y)) \quad ----(16)$$

Each equation shows a sinusoidal fringe pattern. Dividing Eq. (16) by Eq. (15) and calculating its arctangent, we obtain the phase $\theta_x(x,y)$ at any point (x,y), that is,

$$\theta_x(x, y) = \arctan\left[\frac{Im(i_{1,0}(x, y))}{Re(i_{1,0}(x, y))}\right] \quad --(17)$$

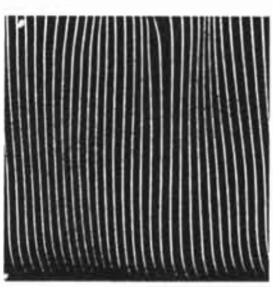

Fig.6 Grating image formed by shefting lines

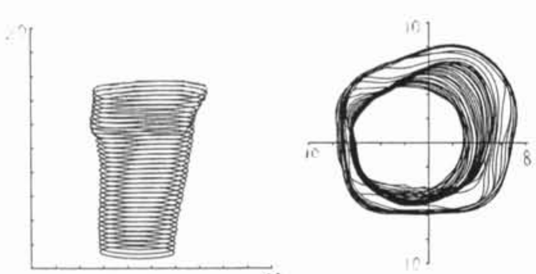

Fig.7 Profile of socket mold

By the same method, we obtain the y-grating phase distribution $\theta_y(x,y)$. As the distribution is assumed linear in small region, the position of the corresponding point (x,y) of a certain phase $(\theta x, \theta y)$ can be calculated in decimal unit, and the phase $(\theta x, \theta y)$ at any given point (x,y) also can be calculated by two-dimensional interpolating.

The phase distribution in the other image input from a CCD camera at a different angle can be computed in the same way. Based on a reference line, we can identify a global phase distribution by phase shifting in each image. Furthermore, if two points in the different images have the same global phase both in X and Y directions, they are identified as the corresponding points. So the corresponding points of different images can be found out by interpolating the phase distribution.

4 APPLICATIONS

Two examples of applications based on the above methods are presented. One is the measurement of the shape of a prosthetic socket mold for an amputated above-knee. The other is the measurement of the shape and strain distribution on a rubber plate.

4.1 Shape Measurement for An Above-knee Socket

An above-knee socket mold is put on a rotatable platform shown in Fig.1. A beam of light is emitted from a laser and pass through an expander which fans it out a vertical line. The line runs onto the socket and is viewed by a CCD camera in a dark room. By rotating the platform at every 12 degree and shifting the recorded laser line image by a 16-pixels correspondently by an image processor NEXUS which is controlled by a NEC PC-9801 computer, a grating image is formed as shown in Figure 6. By calculating the Fourier transform of this grating image, extracting the first harmonic of the distribution, and calculating its inverse Fourier transform, the phase distribution is obtained using the Eq. (14). So, the accurate positions of the strip lines are obtained by calculating and interpolating the phase. And then, the accurate three-dimensional positions on the full surface of the socket mold is analyzed according to the geometrical relation.

The calibration of the system is practiced for the optical distortions and any electrical and mechanical changes. A standard size column whose dimension is known is measured for calibrating the parameter of the system.

Fig.7 shows the profile of the socket mold measured by the above method.

4.2 Shape and Strain Measurement for Rubber Plate

The second application is for a rubber plate that a two-dimensional grating pattern is marked on its surface for finding corresponding points as shown in Figure 8a. It is also put on a rotatable platform shown in Figure 3.

The two-dimensional Fourier spectrum is shown in Fig.8b. Extracting its first harmonics in x- and y-directions respectively by filter. The x-grating and y-grating are separated from the original image respectively. The real part of their inverse Fourier transform is shown in Fig.8c and Fig.8d, and the imaginary part of their inverse Fourier transform is shown in Fig.8e and Fig.8f.

Based on a reference line, the global phase distribution is determined. position (x,y) in different image corresponding to the global phase (ϕ_x,θ_y) can be found out, that is the corresponding point in different image is identified, The exact positions of the cross point of the grating can be found out.

The parameter of the system is readily calibrated by measuring rulers in several directions, Thus the three-dimensional database of the position of grid points is resolved by the stereoscopic relation expressed in Eq.(2) - Eq.(7). Figure 9a shows the measurement result of the shape of the rubber plate.

After deformation, the shape of the rubber plate is measured in the same method, and another database of the profile is formed by the same corresponding points. The strain distributions of the rubber plate is calculated by analyzing those two database of the corresponding points using Eq.(8) - Eq.(10). Fig.9b shows the measurement result of the normal strain (ε_x) distribution corresponding to the Fig.9a.

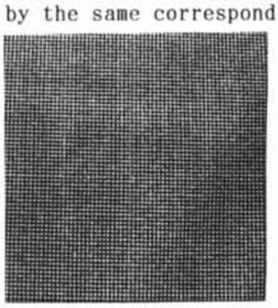

Fig.8a Image of rubber plate

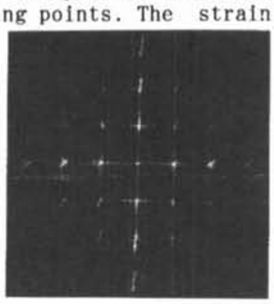

Fig.8b Fourier spectra of Fig.8a

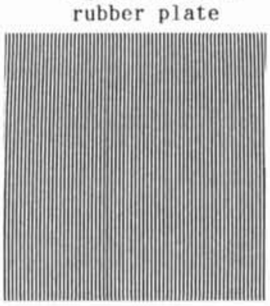

Fig.8c Real part in x-directional

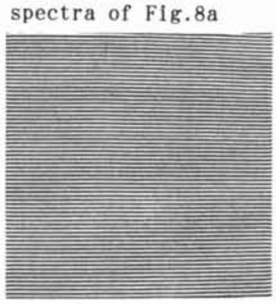

Fig.8d Real part in y-directional

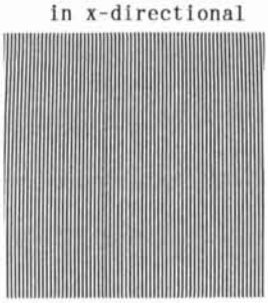

Fig.8e Imaginary part in x-directional

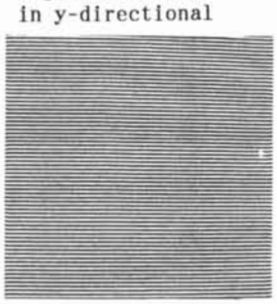

Fig.8f Imaginary part in y-directional

5 CONCLUSION

A simple, accurate image analysis method for measuring the shape and strain distribution of a 3D object is developed by Using Fourier transform grid method. Using it, the two dimensional grating can be easily separated to x- and y-grating based on its orthogonality, and the decimal pixel unit measurement can be realized by calculating and interpolating phase distribution of the image.

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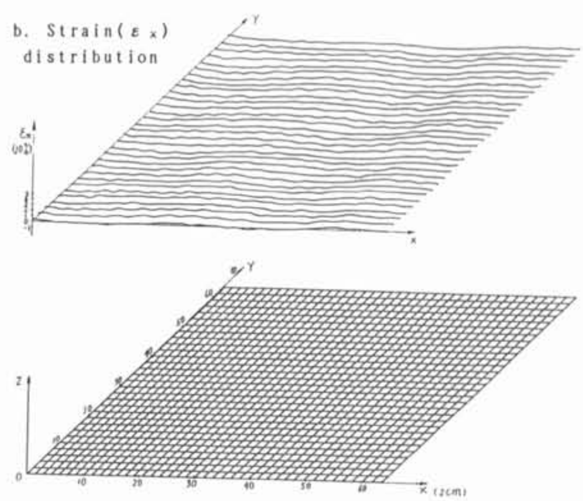

Fig.9 Shape and strain of rubber plate by measured