By substituting Equation (7) into Equations (5) and (6), the following two linear independent equations is derived:

$$s_g = (m\frac{i_g^1}{i_r^1})s_r + i_g^1 c \quad , \quad s_g = (m\frac{i_g^2}{i_r^2})s_r + i_g^2 c \qquad (8)$$

Each equation means an expanded Illumination line whose r axis is scaled by the factor of i_r and whose g axis is scaled by the factor of i_g. The surface color $\mathbf{s}$=(s_r, s_g) becomes the intersection of two generated lines.

3 Robust Estimation

3.1 Instability

While the estimation using the *Illumination line* is simple and elegant, we discovered that it is significantly sensitive to input errors. When the input chromaticities include a small amount of errors, two generated lines (Equation (8)) will be incorrect. As a result, the intersection, which represents the surface color, will deviate significantly from the correct one, especially when the gradients of two lines are similar.

We have analytically investigated the effect of input errors[2] and summarized them in Table 1. The important value in the table is

$$W \approx \frac{1}{\left(1 - \frac{e_r^1/e_g^1}{e_r^2/e_g^2}\right)} \qquad (9)$$

where we assumed $1+\Delta i_*^1/i_*^1 \cong 1$. The value W magnifies the input errors when two illumination colors are similar. W approximately commonly appears in Table 1. Thus, any kind of input errors will be equally affected by the value of W. We simulated the numerical value of W using blackbody radiation for two illuminants, and found that if the temperature difference between those two illuminants is less than 200 mired[1], the estimation error always becomes larger than the input error.

We have also analytically derived that the estimation error increases approximately linearly against the multiplication of the sum of input errors Δ^{input} and the color difference parameter W. This is illustrated in Figure 1. The vertical axis is the CIE-LAB color difference ($\equiv \Delta E*_{ab}$) between the estimated illumination color $\mathbf{e}_{est}^1$ and the true illumination color $\mathbf{e}_{true}^1$. Figure 1 shows that $\Delta^{input}W$ needs to be less than 0.2 on average, if the estimation error have to be less than 10 errors.

We have investigated how much amount of input errors would be included on average by simulating numbers of colors with many kinds of relfectance and illuminations using the sensitivity of SONY DXC-9000. We excluded the details because of limited space, but the average valuve was about Δ^{input}=0.05. Based on the analysis, the preferable color temperature difference between two illumination colors is more than 100 mired ($W = 4$).

[1] mired= $10^6/K$. 200 mired difference corresponds to the difference between 3300K and 10000K. This correponds the color range of natural outdoor illuminations.

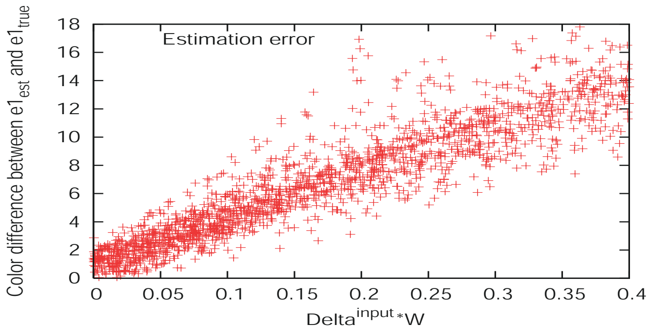

Figure 1: The estimation error in CIE-LAB color space ($\equiv \Delta E*_{ab}$) against $\Delta^{input}W$. The estimation error increases approximately linearly as $\Delta^{input}W$ increases.

3.2 Illumination Line Segment

From the analysis in 3.1, we have found that the estimation using the *Illumination line* may produce a large estimation error. However, the error can be relatively suppressed if we restrict the illumination color to exist in the reasonable range of the color space. Based on this idea, we propose the following method for detecting the estimation error and reducing it as much as possible.

Since the possible range of outdoor illumination colors is finite, the Illumination line is actually a line segment in the real world, which we call the *Illumination line segment*. We measured outdoor illuminations in Tokyo during a day, and determined the rage as from 3500K to 12000K. We fitted a line to the Planckian locus with this range, and calculated the gradient and the intercept of the Illumination line segment.

Two lines generated by Equation (8) become two line segments. The surface color is the intersection of them. The important point is that *when the intersection divides a line in p:1-p, the illumination color* $\mathbf{e}^1$ *divides the Illumination line segment in p:1-p* as illustrated in Figure 2. Let one line segment generated from $\mathbf{i}^1$ be Line 1, and the other line segment generated from $\mathbf{i}^2$ be Line 2. When the intersection internally divides Line 1 in p:1-p, the illumination color $\mathbf{e}^1$ would be the point that divides the Illumination line segment in p:1-p.

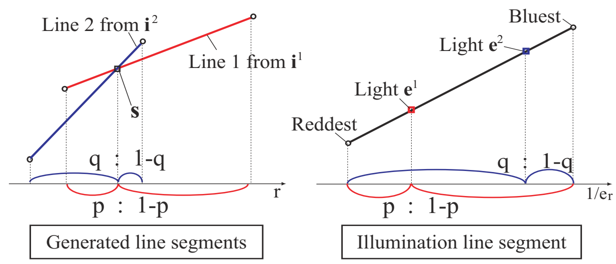

Figure 2: Lines 1 and 2 are generated by $\mathbf{i}^1$ and $\mathbf{i}^2$. When the intersection $\mathbf{s}$ divides Line 1 in p:1-p, the light color $\mathbf{e}^1$ divides the Illumination line segment in p:1-p.

Table 1: The value of estimation errors when the inputs include small errors.

Input error	Estimation error $\Delta s_r/s_r$	Estimation error $\Delta s_g/s_g$
$\hat{i_r^1} = i_r^1 + \Delta i_r^1$	$\frac{\Delta i_r^1}{i_r^1} \frac{1}{1-\left(1+\frac{\Delta i_r^1}{i_r^1}\right)\frac{e_r^1/e_g^1}{e_r^2/e_g^2}}$	$\frac{\Delta i_r^1}{i_r^1} \frac{1}{1-\left(1+\frac{\Delta i_r^1}{i_r^1}\right)\frac{e_r^1/e_g^1}{e_r^2/e_g^2}} \frac{i_r^1}{i_g^1} \frac{i_g^2-i_g^1}{i_r^2-i_r^1}$
$\hat{i_g^1} = i_g^1 + \Delta i_g^1$	$\frac{\Delta i_g^1}{i_g^1} \frac{-1}{\left(1+\frac{\Delta i_g^1}{i_g^1}\right)-\frac{e_r^1/e_g^1}{e_r^2/e_g^2}} \frac{i_g^2}{i_r^2} \frac{i_r^2-i_r^1}{i_g^2-i_g^1}$	$\frac{\Delta i_g^1}{i_g^1} \frac{-1}{\left(1+\frac{\Delta i_g^1}{i_g^1}\right)-\frac{e_r^1/e_g^1}{e_r^2/e_g^2}} \frac{i_r^1}{i_g^1} \frac{i_g^2}{i_r^2}$

3.3 Detecting and Correcting Errors

Detection When the intersection of two generated lines does not exist on those line segments, the reason would be: (1) The color temperature of either $\mathbf{e}^1$ or $\mathbf{e}^2$ is outside the range of the Illumination line segment. (2) The lines are deviated because of the input errors. Statistically the first case rarely occurs. Thus, the reason would be the second; the inputs have errors.

Suppose that we generate a line segment from an image chromaticity, and let the coordinates of its start and end points be $(l^1_{r\min}, l^1_{g\min})$ and $(l^1_{r\max}, l^1_{g\max})$. Let the other's be $(l^2_{r\min}, l^2_{g\min})$ and $(l^2_{r\max}, l^2_{g\max})$. The intersection point is (s_r, s_g). The intersection exists on both line segments if,

$$l^1_{r\min} \le s_r \le l^1_{r\max} \quad \mathit{and} \quad l^2_{r\min} \le s_r \le l^2_{r\max}. \qquad (10)$$

The above condition may be rewritten as follows by using the g-coordinate.

$$l^1_{g\min} \le s_g \le l^1_{g\max} \quad \mathit{and} \quad l^2_{g\min} \le s_g \le l^2_{g\max}. \qquad (11)$$

Correction When the intersection does not exist on both line segments, there exist input errors. Therefore, the method selects one of the input image chromaticities and adjusts it so that the intersection will be on both lines. Since an input image chromaticity determines the gradient and the intercept of the generated line, it can change the position of the intersection.

Input image chromaticities consist of four values i_r^1, i_g^1, i_r^2, i_g^2, but we only adjust one of them. This is for algorithmic reason. If we allow two values to change, one can move the intersection to an arbitrary position in many ways. How to select one from four values is explained below.

The correction value can be calculated using Equation (10) or (11). When the value i_g^2 is selected, for instance, Equation (10) determine the range in which the adjusted value $\tilde{i_g^2}$ should be. The r-coordinate of the surface color s_r can be expressed as

$$s_r = \frac{i_r^1 i_r^2 (i_g^2 - i_g^1)}{i_r^2 i_g^1 - i_r^1 i_g^2} \frac{c}{m} \qquad (12)$$

from Equation (8). By substituting the last equation into Equation (10), the range for the adjusted value $\tilde{i_g^2}$ can be calculated:

$$i_g^1 \frac{i_r^2 (l^*_{r\min} \frac{m}{c} + i_r^1)}{i_r^1 (l^*_{r\min} \frac{m}{c} + i_r^2)} \le \tilde{i_g^2} \le i_g^1 \frac{i_r^2 (l^*_{r\max} \frac{m}{c} + i_r^1)}{i_r^1 (l^*_{r\max} \frac{m}{c} + i_r^2)} \quad * = \{1, 2\}$$

where it assumes $i_r^2 i_g^1 - i_r^1 i_g^2 > 0$, which means the chromaticity $\mathbf{i}^2$ is assumed to be redder than $\mathbf{i}^1$. Otherwise the symbol $<$ must be replaced by $>$. The current value i_g^2 will be adjusted to the nearest $\tilde{i_g^2}$ that satisfies the last equation.

In the case where we select i_r^2 for adjustment, the range for i_r^2 can be calculated in a similar way.

$$i_r^1 \frac{i_g^2 (l^*_{g\max} \frac{1}{c} - i_g^1)}{i_g^1 (l^*_{g\max} \frac{1}{c} - i_g^2)} \le i_r^2 \le i_r^1 \frac{i_g^2 (l^*_{g\min} \frac{1}{c} - i_g^1)}{i_g^1 (l^*_{g\min} \frac{1}{c} - i_g^2)} \quad * = \{1, 2\}$$

where it assumes $i_r^2 i_g^1 - i_r^1 i_g^2 > 0$ and $l^*_{g\min} \frac{1}{c} - i_g^2 > 0$.

Note that the method cannot handle the following three cases; (1) $i_r^2 i_g^1 - i_r^1 i_g^2 > 0$ does not hold after adjusting i_g^2 or i_r^2. (2) There is no common range in Equation (10), or in Equation (11). (3) The adjusted value such as $\tilde{i_g^2}$ becomes equal to the other input value i_g^1. This is a rare case.

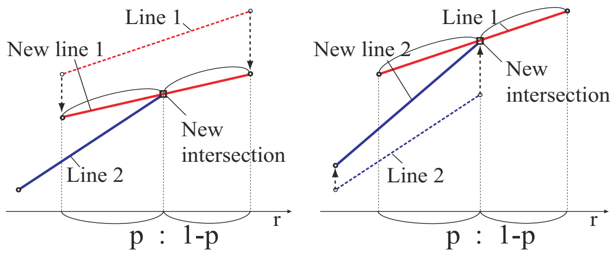

Figure 3: Left: The case of adjusting i_g^1. Right: The case of adjusting i_g^2. The ratio p does not change in both cases.

Selecting an Input There is an issue of selecting one from four image chromaticity values i_r^1, i_g^1, i_r^2, i_g^2 to be adjusted. We have found that there is no difference between selecting input $\mathbf{i}^1$ and selecting input $\mathbf{i}^2$. However, a different estimation results depending on which channel to select, r or g. Therefore, we calculate the effectiveness of each channel and select the one with greater effectiveness.

Selecting input $\mathbf{i}^1$ or selecting input $\mathbf{i}^2$ would make no difference in illumination color estimation. Figure 3 illustrates this. The left of Figure 3 is the case of selecting i_g^1, and the right is the case of selecting i_g^2. Whichever we adjust i_g^1 or i_g^2, the new intersection would internally divide each line segment in the same ratio. As mentioned in 3.2, the ratio determines the illumination colors and thus the estimated results would be unchanged in both cases.

On the other hand, a different estimation results depending on which channel to select, r or g. Therefore, first, the method checks if the inputs match the cases stated in 4.3 that the method cannot handle. If the inputs do not, then it calculates the effectiveness of each channel and select the one that has greater effectiveness. We defined effectiveness α as:

$$\alpha = \left| \frac{\left(i_r^2 - i_r^1\right)}{\left(i_g^2 - i_g^1\right)} \frac{i_g^2}{i_r^2} \right| \tag{13}$$

The above equation is derived from Table1. It assumes that the input $\mathbf{i}^1$ is going to be adjusted. If α is larger than 1, a small difference in channel g affects the estimation compared to channel r and therefore the method selects channel g to be adjusted. Otherwise, channel r will be selected.

4 Experimental Results

4.1 Method

Images of the GretagMacbeth ColorChecker were taken under five illuminants using SONY DXC-9000 progressive 3 CCD digital cameras by setting its gamma correction off. The details of five illuminants are shown in Talbe 2. In order to make the color temperature difference between two illuminants relatively large, we used L1 and L2 for redder illuminants, and used L3 through L5 for bluer illuminants (the minimum color difference: 78 mired, the maximum: 185 mired.) Cutting out the images of 18 surface colors of Machbeth chart, we prepared 18×5 color patch images (each was resized to 70×70 pixels.)

Having selected a pair of color patch images whose surface color is identical, we randomly chose a pixel from each images and estimated their illumination chromaticities. This was repeated 200 times for a pair. The same was done with the previous method[1] for comparison. The total combination number of two illuminants and a surface was $2 \times 3 \times 18 = 108$.

Table 2: Details of five Illuminants used in experiments.

	Details	CCT
L1	2003/8/22 17:30, Tokyo, Outdoor illumination, Fine day	3539K
L2	2003/8/22 16:30, Tokyo, Outdoor illumination, Fine day	4083K
L3	2003/7/14 14:30, Tokyo, Outdoor illumination, Cloudy day	5991K
L4	2003/7/14 18:20, Tokyo, Outdoor illumination, Cloudy day	7083K
L5	2003/7/14 18:55, Tokyo, Outdoor illumination, Cloudy day	10266K

4.2 Results

From the results of aforementioned 108 experiments, we obtained the estimation error histograms of both the proposed method and the previous method. They are shown in Figure4. Here, the estimation error means the difference between the true illumination chromaticity and the estimated illumination chromaticity in CIE-LAB color space. The proposed method could suppress both the average and the maximum error values. The average estimation errors of the proposed method were 5.2 (redder illuminants) and 3.7 (bluer illuminants.) Those of the previous method were 11.3 (redder illuminants) and 13.1 (bluer illuminants.) There were 10 data whose error were larger than 20 in the previous method's results, and 0 in the proposed method's results.

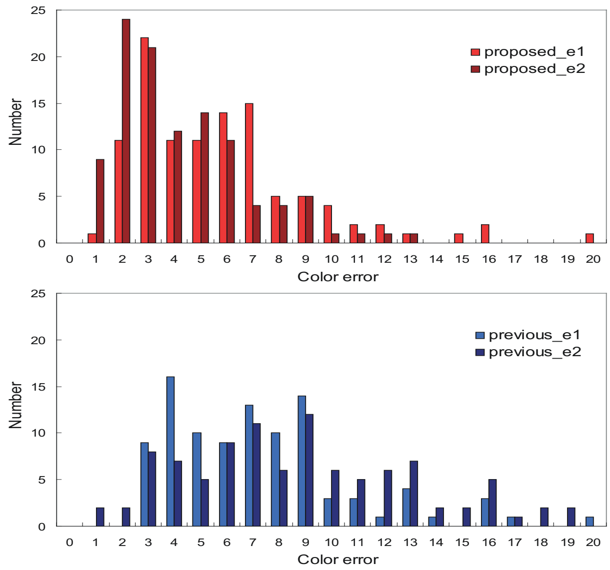

Figure 4: Histograms of color errors. Top: Results of the proposed method. Bottom: Results of the previous method. Color error means the color difference between the true illuminant's color and the estimated illuminant's color in CIE-LAB space.

5 Conclusion

We have proposed a method to stably estimate surface and illumination chromaticities using illumination color variance. Having analyzed the effect of input erros on the previous method, we developed a method that is robust and accurate by considering the possible range of outdoor illumination colors, which we call the "Illumination line segment". The experimental evaluation showed the effectiveness of our method.

The remaining problem is the improvement of the estimation accuracy.

Acknowledgement

This work was, in part, supported by Ministry of Education, Culture, Sports, Science and Technology, under the program, "Development of High Fidelity Digitization Software for Large-Scale and Intangible Cultural Assets."

References

[1] G.D. Finlayson, et al. Color constancy under varying illumination. *Proc. IEEE Int'l Conf. Computer Vision*, pp.720-725, 1995.

[2] R.Kawakami, et al. Consistent Surface Color for Texturing Large Objects in Outdoor Scenes. In *Proc. of 10th IEEE Int'l. Conf. on Computer Vision*, 2005.

[3] D.B. Judd, et al. Spectral distribution of typical daylight as a function of correlated color temperature. *J. Optical Soc. Am.*, 54(8):1031-1040, 1964.

[4] S. Tominaga, B.A. Wandell. Natural scene-illuminant estimation using the sensor correlation. In *Proc. IEEE*, 90(1):42-56, 2002.