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Propagation of Uncertainty in Landmark Based Self-localization of Autonomous Mobile Robots

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Abstract

This paper presents uncertainty propagation in landmark based position estimation methods. Analysis of two methods has been carried out where robot position is estimated by detecting one or two globally distinct features using a pivoted stereo vision system. We make a basic assumption about error in estimating point features in camera images and propagate it into robot position estimate using first order approximation of non-linear functions. Simulation results illustrate the performance of the method.

1 Introduction

Robot pose maintenance is a basic requirement of autonomous navigation. To estimate its pose a robot normally relies on its sensors to gather information about distinctive features in its environment. Error free measurements will result in perfect localization. However, measurements are never perfect which result in an uncertain robot position estimate. Apart from measurement errors there could be error in feature identification and matching with the world map. Therefore, in addition to knowing position of the robot it is also important to measure reliability of this estimate.

Matthies and Shafer [1] use a stereo vision system to estimate translation and rotation between two 3D sets of points. The robot position is estimated as a succession of each transformation between frames. Similarly, Kriegman et al. [2] use a stereo vision system for robot navigation in indoor environments. Robot position is tracked, based on information gathered as features move between each successive motion of the robot. Robot position uncertainty is calculated using first order approximation.

Fuzzy logic based methods can be used to account for error in odometry and external sensors and to represent uncertainty in robot location. Buschka et al. [3] adopt this technique for localization of Sony Aibo robots using lines and color marking in the environment, whereas, Herrero-Perez et al. [4] use corner formed by field lines. Similarly, fuzzy temporal rules have been applied to detect doors in ultrasonic data [5] and Demirli and Türksen [6] use fuzzy sets to model sonar data and estimate robot position by using fuzzy triangulation.

There are approaches that represent observation error as tolerances. Atiya and Hager [7] use a geometric tolerance to express observation uncertainty. They use stereo vision based extraction of vertical edges in the environment for localization. Similarly, Boley et al. [8] propose an alternate solution to extended Kalman filter. They use recursive total least squares algorithm to obtain estimate of the robot position.

In this work, robot observation is based on identification of globally distinct landmarks in robot coordinates system using binocular stereo vision system. The robot position is then calculated using landmark location in robot and world coordinate system as input. We assume that the non-linear models can be adequately represented by the first two components of the Taylor series expansion around the estimated observation. However, as noted by Kriegman et al. [2], the linearization of the perspective transformation do not hold for distant point correspondence as the disparity decreases and higher order terms will dominate. We are mainly interested in analyzing location uncertainty that arises from sensor imperfections. We assume that error in landmark location in the global map and correspondence analysis can be ignored. Our work differs from earlier works in a way that we assume that the random error in pixels location in each image of the stereo pair is Gaussian and then propagate it to position

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estimation of the robot. The balance of the paper is structured as follows: Section 2 reviews two methods of position estimation and presents uncertainty analysis. Experimental results are presented in Section 3. The paper is concluded in Section 4

2 Uncertainty analysis

We assume that the robot's motion is two dimensional where pose of the robot has 3 degrees of freedom i.e. $\mathbf{p} = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$. The global coordinate system is represented by x and y axis, the robot coordinate system by X and Y axis and the image coordinate system by u and v axis. The u and v axes of the image plane are in opposite direction of Y and Z axes respectively. Assuming identical cameras, parallel image planes and aligned epipolar lines a point $\mathbf{P} = \begin{bmatrix} X & Y & Z \end{bmatrix}^T$ in robot coordinate system can be related to its projections $\mathbf{p}_{il} = \begin{bmatrix} u_l & v_l \end{bmatrix}^T$ and $\mathbf{p}_{ir} = \begin{bmatrix} u_r & v_r \end{bmatrix}^T$ in the left and right images under perspective transformation as follows [9]:

$$\mathbf{P} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{f}(\mathbf{p}_{il}, \mathbf{p}_{ir}) = \begin{bmatrix} x_c + \frac{fb}{u_l - u_r} \\ \frac{-b}{2} \frac{u_l + u_r - 2o_u}{u_l - u_r} \\ \frac{-b(v_r - o_v)}{u_l - u_r} \end{bmatrix} \quad (1)$$

where $[o_u \; o_v]^T$ is the image center, b is the baseline of the stereo vision system, f focal length of both cameras and x_c is the distance from the center of the robot to the cameras.

2.1 Single landmark based localization

Robot position $\mathbf{p} = \begin{bmatrix} x & y \end{bmatrix}^T$ can be calculated using observation X and Y with respect to landmark $\mathbf{p}_l = \begin{bmatrix} x_l & y_l \end{bmatrix}^T$ and absolute orientation θ as given by (2) [9]:

$$\mathbf{p} = \begin{bmatrix} x \\ y \end{bmatrix} = \mathbf{g}(\begin{bmatrix} u_l \\ u_r \\ \theta \end{bmatrix}) = \begin{bmatrix} x_l - rc \\ y_l - rs \end{bmatrix} \quad (2)$$

where $s = \sin(atan2(^Y/_X) + \theta)$, $r = \sqrt{X^2 + Y^2}$ and $c = \cos(atan2(^Y/_X) + \theta)$.

The imperfection of the input quantities result in an uncertain position estimate as shown in Fig. 1. We assume that error in input vector is zero mean Gaussian with the following covariance:

$$\mathbf{\Sigma}_i = \begin{bmatrix} \sigma_{uu}^2 & 0 & 0 \\ 0 & \sigma_{uu}^2 & 0 \\ 0 & 0 & \sigma_{\theta\theta}^2 \end{bmatrix} \quad (3)$$

where σ_{uu}^2 is the variance of u_l or u_r and $\sigma_{\theta\theta}^2$ is the variance of robot absolute orientation. All three components of the input vector are not correlated to each other. Additionally, due to the identical camera assumption the error distribution of u_l and u_r are supposed to be identical. This error is propagated into position estimate by the transformation (2). The system given by (2) is nonlinear and the resulting error distribution will not be a Gaussian. However, we assume that it can be adequately represented by the first two terms of Taylor series expansion around estimated input, $\widehat{\mathbf{i}}$. We proceed as follows:

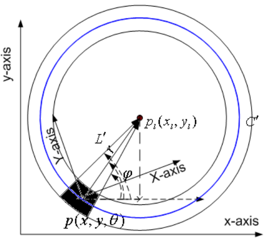


Figure 1: Error in landmark location and robot orientation results in an uncertain pose estimation

$$\mathbf{p} = \mathbf{g}(\widehat{\mathbf{i}}) + \mathbf{J}_i \widetilde{\mathbf{i}} + \dots \quad (4)$$

where $\widehat{\mathbf{p}} = \mathbf{g}(\widehat{\mathbf{i}})$ is the position estimate, $\widetilde{\mathbf{p}} \approx \mathbf{J}_i \widetilde{\mathbf{i}}$ its error and $\mathbf{J}_i$ is the jacobian of $\mathbf{p}$ with respect to $\mathbf{i}$ evaluated at its estimated value $\widehat{\mathbf{i}}$. $\mathbf{J}_i$ is having the following elements

$$\begin{aligned} {}^{\partial x}/_{\partial u_l} &= {}^1/_{rd^2}[u_{ro}(bsX - cbY) + fb(sY + cX)] \\ {}^{\partial x}/_{\partial u_r} &= {}^{-1}/_{rd^2}[u_{lo}(bsX - cbY) + fb(sY + cX)] \\ {}^{\partial x}/_{\partial \theta} &= rs \\ {}^{\partial y}/_{\partial u_l} &= {}^{-1}/_{rd^2}[u_{ro}(bcX + sbY) + fb(cY - sX)] \\ {}^{\partial y}/_{\partial u_r} &= {}^1/_{rd^2}[u_{lo}(bcX + sbY) + fb(cY - sX)] \\ {}^{\partial y}/_{\partial \theta} &= -rc \end{aligned}$$

where $d = u_l - u_r$, $u_{ro} = u_r - o_u$ and $u_{lo} = u_l - o_u$. From (4) we derive expression for the covariance matrix of position estimate as follows:

$$\mathbf{\Sigma}_p = E\{\widetilde{\mathbf{p}}\widetilde{\mathbf{p}}^T\} = \mathbf{J}_i \mathbf{\Sigma}_i \mathbf{J}_i^T$$

2.2 Location estimation using two landmarks

When the robot orientation estimate is not reliable or not available it needs to estimate location of two globally known landmarks in its coordinate system [10]. With this information robot location is constrained to the intersection of two circles as shown in Fig. 2 and given by the following expression:

$$\mathbf{p} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = \begin{bmatrix} {}^{(-D \pm \sqrt{D^2 - 4CE})}/_{2C} \\ A + B\,{}^{(-D \pm \sqrt{D^2 - 4CE})}/_{2C} \\ atan2({}^{y_{l1} - y}/_{x_{l1} - x}) - atan2({}^{Y1}/_{X1}) \end{bmatrix} \quad (5)$$

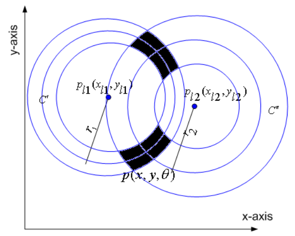


Figure 2: Range estimation error with respect to landmarks constrains the robot location to shaded area instead single point of intersection of the circle

where $A = \frac{r_1^2 - r_2^2 + x_{l2}^2 - x_{l1}^2 + y_{l2}^2 - y_{l1}^2}{2a}$, $B = \frac{x_{l1} - x_{l2}}{a}$, $C = B^2 + 1$, $D = 2AB - 2y_{l1}B - 2x_{l1}$, $E = A^2 + x_{l1}^2 + y_{l1}^2 - 2y_{l1}A - r_1^2$ and $r_1 = \sqrt{X1^2 + Y1^2}$, $r_2 = \sqrt{X2^2 + Y2^2}$, $a = y_{l2} - y_{l1}$. The subscripts 1 and 2 differentiate between quantities related to the two landmarks.

The input vector $\mathbf{i} = \begin{bmatrix} u_{l1} & u_{r1} & u_{l2} & u_{r2} \end{bmatrix}^T$ has four components which correspond to the location of the two landmarks in the camera images. The imperfection in its estimation is propagated into robot location using (5) which result in an uncertain position estimate as illustrated in Fig. 2. We model this imperfection with a zero mean Gaussian having the following covariance matrix:

$$\mathbf{\Sigma}_i = \sigma_{uu}^2 \mathbf{I}_{4\times4} \tag{6}$$

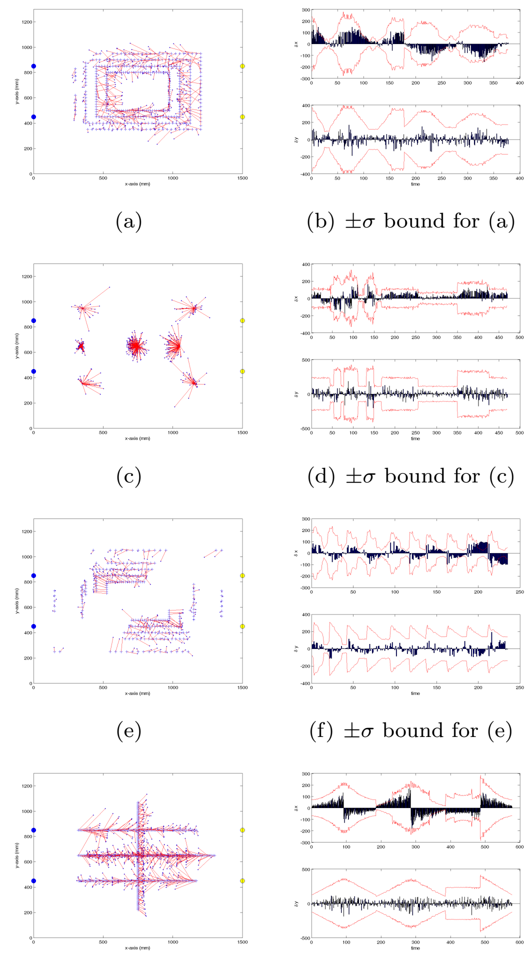
where $\mathbf{I}_{4\times4}$ is 4×4 identity matrix. Using the same principles as adapted in the previous section we arrive at the following expression for the covariance matrix of position estimate using the new method:

$$\mathbf{\Sigma}_p = \mathbf{J}_i \mathbf{\Sigma}_i \mathbf{J}_i^T \tag{7}$$

3 Experimental results

We investigate the performance of our algorithm in simulation. In total 25 trials each consisting of 100 steps were conducted. These trials are further grouped into four categories. First the single landmark method was tested and then the experiment was repeated using two landmarks. At every step the robot calculates it position and its uncertainty if it finds the minimum required features.

Experimental results of single landmark based position estimation are shown in Fig. 3. The first category consisting of four trials is shown in Fig. 3(a). Here the robot follows rectangular paths of different dimensions with its orientation fixed at 0 ° or 180 °. To compare position deviation with its corresponding $\pm\sigma$ error bound all measurements in each category are stacked together and shown next to it. For example Fig. 3(b) shows error in all position estimates of Fig. 3(a). Resulting error in robot position is shown by black bars while the red curve give the $\pm\sigma$ uncertainty bound. We see from Fig. 3(b), 3(d), 3(f) and 3(h) that the error is well bounded. Fig. 3(c) illustrates a rotation only scenario. Between each consecutive steps there is only a small change in robot orientation whereas its position remains fixed. Motion along rectangular paths of different dimension and rotation of 360 °in each trial is shown in Fig. 3(e). This category consists of 5 trials. Fig. 3(g) illustrate the last category consisting of 6 trials. Here the robot is following linear paths with its orientation fixed along the x-axis.



(a) (b) $\pm\sigma$ bound for (a)

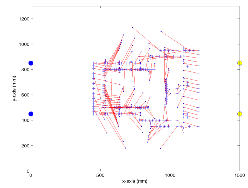
(c) (d) $\pm\sigma$ bound for (c)

(e) (f) $\pm\sigma$ bound for (e)

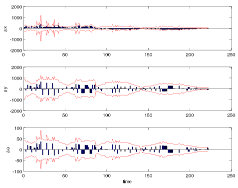
(g) (h) $\pm\sigma$ bound for (g)

Figure 3: Single landmark position estimation. Robot orientation noise is additive zero mean and 12.25 °variance (a) motion along rectangular paths with no rotation (c) fixed point rotation (e) motion and rotation (g) linear motion with no rotation

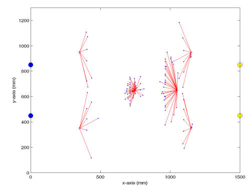
Results for the second method are shown in Fig. 4. True positions are marked with a (+), while calculated positions are marked with a $(\cdot)$. The line segments show the difference between the calculated and true position. The small circles on the left and right sides indicate the four points used as landmarks in this study. In all of these experiments image resolution and stereo baseline was set at 320 × 240 pixels and 3cm respectively. The experiment was conducted with σ^2_{uu} and $\sigma^2_{\theta\theta}$ values of 0.9 and $pi/20$ respectively.



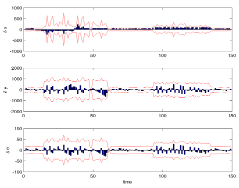
(a) Motion only



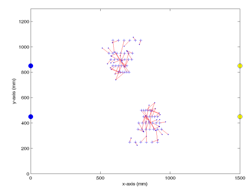
(b) $\pm\sigma$ bound for (a)



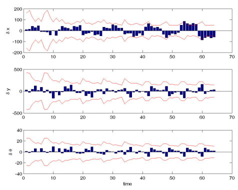
(c) Fixed point rotation



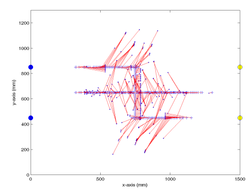
(d) $\pm\sigma$ bound for (c)



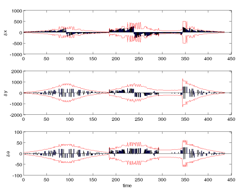
(e) Motion and rotation



(f) $\pm\sigma$ bound for (e)



(g) Linear motion with no rotation



(h) $\pm\sigma$ bound for (g)

Figure 4: Robot pose is estimated at fewer locations as compared to the first method due to the difficulty of simultaneous acquisition of two landmarks

As can be seen from Fig. 3 and Fig. 4 that due to high range error robot location error is high when it is estimated with respect to distant feature points. This is due to the fact that we use a narrow baseline stereo and low resolution images. Furthermore, the appearance of the landmark features varies with the view point. This raise in location error with distance from landmark features is properly captured by a corresponding increase in uncertainty.

4 Conclusion

In this paper we presented uncertainty analysis using first order error approximation of explicit functions. Simulation results show that the method can adequately represent uncertainty that arise from image quantization error. However, the method fails to handle gross segmentation errors and mismatches of landmarks. Global position estimate and its covariance matrix is required to initialize extended Kalman filter for tracking robot position. Future work include completion of the real-world tests and incorporating these results in an extended Kalman filter for robot position tracking.

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