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Noisy Image Segmentation Based on a Level Set Evolution

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Abstract

In this paper, we propose a new hybrid model for active contour image segmentation, which is able to segment non-uniform noisy images efficiently. The model is a combination between the classical active contour based on the image gradient and the mean curvature moving technique. The efficiency is achieved by de-noising the image using log-Gabor filter then using a hybrid model to segment the noise free image. The proposed model has three main advantages over the pervious models and other traditional segmentation techniques. First, a significantly larger time step can be used for numerically solving the evolution PDE, and therefore speed up the curve evolution. Second, the model can be used to segment the image in the presence of high or low noise. Third, the proposed model can be used for segmenting a single object or grouping multi-objects in the image. We will present various experimental results on natural and synthetic images which demonstrate the power of the proposed method for segmentation of noisy image.

1. Introduction

Segmentation of noisy image is still a challenging problem and its successfully solution is either based on simple intensity threshlding or by model based deformation of templates. The former implies that the structures are well separated by unique intensity patterns, whereas the latter requires model templates characteristics for the shape class. A wide variety of approaches for image segmentation procedures are documented in the general image processing literature and many successful algorithms have been proposed and developed [3].

In recent years, the theory of the partial differential equation (PDE) has been applied very successfully to image segmentation and image smoothing [5, 6]. The basic idea is to represent contours as the zero level set of an implicit function defined in a higher dimension, usually referred as the level set function and evolve the level set function according to PDF. The level set method is a general technique for evolving curves or surfaces that may undergo complex topological changes such as merging and pinching. The level set equation is given by:

$$\frac{\partial \phi}{\partial t} + F\,|\nabla \phi| = 0 \qquad (1)$$

where, ϕ is the level set function and F is a speed function. This equation is used to keep track of the interface location as the set points where $\phi = 0$. The interior and exterior are then designated by the points where $\phi \le 0$ and $\phi > 0$ respectively.

The Osher-Sethian [7] Level set method considers evolving fronts in an implicit form. It is a numerical method that works in a fixed coordinate system and takes care of the topological changes of the evolving interface. Their active contour model written in the level set formulation is given by:

$$\frac{\partial \phi}{\partial t} = \left(\upsilon g(x,y) + div\left(g(x,y) \frac{\nabla \phi}{|\nabla \phi|} \right) \right) |\nabla \phi| \qquad (2)$$

The first part of the right hand side represents the weighted area minimization term that yields a constant velocity where υ is a constant. The second part is the waited length term.

In this model, an edge detector, depending on the gradient of the initial image, is used to stop the evolving curve on the boundary of the desired object. Usually, this is a positive and regular edge function $g(|\nabla \boldsymbol{I}|)$ which decreases such as $\lim_{t\to\infty} g(t) = 0$.

$$g\left(\left|\nabla \boldsymbol{I}(x,y)\right|\right) = \frac{1}{1 + \left|\nabla \boldsymbol{I}_s(x,y)\right|} \qquad (3)$$

where $\boldsymbol{I_s(x,y)}$ is the smooth version of image $\boldsymbol{I(x,y)}$ given by the convolution of $\boldsymbol{I(x,y)}$ with the Gaussian. This classical contour has a main drawback that depends on the gradient of the image to stop the curve evolution; therefore, this model can detect only objects with edges defined by gradient. Moreover, if the image $\boldsymbol{I(x,y)}$ is noisy, then the isotropic smoothing Gaussian has to be strong, which will smooth the edges too. Furthermore, the function ϕ should be re-initialized periodically during the evolution. There has been copious literature on re-initialization methods [8], and most of them are variants of the above PDE-based method. The re-initialization process is quite complicated, expensive, and has subtle side effects. Moreover, most of the level set methods are fraught with their own problems, such as when and how to re-initialize the level set function to a signed distance function. The new variational formation suggested by C.Li et al [9] overcomes the last difficulty by adding an internal energy term that penalizes the deviation of the level set function from a signed distance function.

Based on the Mumford-Shah minimal partition functional, Chan and Vese [1] proposed a new level set model for active contours to detect objects whose boundaries are not necessarily defined by the gradient. They introduce an energy function given by:

$$F\left(\phi, c_1, c_2\right) = \mu \int_{\Omega} \delta_{\varepsilon}\left(\phi\right)\left|\nabla \phi\right| d\Omega + \upsilon \int_{\Omega} H_{\varepsilon}\left(\phi\right) d\Omega$$
$$+ \lambda_1 \int_{\Omega} \left|\boldsymbol{I} - c_1\right|^2 H_{\varepsilon}\left(\phi\right) d\Omega \qquad (4)$$
$$+ \lambda_2 \int_{\Omega} \left|\boldsymbol{I} - c_2\right|^2 \left(1 - H_{\varepsilon}\left(\phi\right)\right) d\Omega$$

with the unknown constants c_i representing the main intensity value of the region labelled *i* and defined by:

$$c_1\left(\phi\right) = \frac{\int_{\Omega} \boldsymbol{I}\, H_{\varepsilon}\left(\phi\right) d\Omega}{\int_{\Omega} H_{\varepsilon}\left(\phi\right) d\Omega}, \quad c_2\left(\phi\right) = \frac{\int_{\Omega} \boldsymbol{I}\left(1 - H\left(\phi\right)\right) d\Omega}{\int_{\Omega} \left(1 - H\left(\phi\right)\right) d\Omega}, \qquad (5)$$

where $H_{\varepsilon}\left(\phi\right)$ and $\delta_{\varepsilon}\left(\phi\right)$ are the regularized version of Heaviside and Dirac function respectively (see e.g. [10] for numerical approximation of H_{ε} and δ_{ε}.

Although this model has many advantages over the traditional one that did not depend on the gradient of the objects, its main drawback is computationally expensive to solve the proposed non-linear parabolic partial differential equation. Furthermore, if the noise is not modelled as a Gaussian ("normal"), or uniform, the segmentation of the image could not be performed. In order to overcome this speed limitation and the problem in segmentation of the non-uniform noise images, we propose a new hybrid model that benefits from the simplicity and efficiency of the classical one in detecting edges while preserving the robustness of the Chan and Vese model. This model enables to eliminate noise as much as possible without smoothing edges under suitable pre-processing.

In this paper, we present a new active contour model that enables to segment the non-uniform noise image efficiently. The efficiency is achieved by de-noising the image using log Gabor filter then using a hybrid model to segment the image. The proposed model has three main advantages over the pervious models and other traditional segmentation techniques. First, a significantly larger time step can be used for numerically solving the evolution PDE, and therefore speed up the curve evolution. Second, the model can be used to segment the image in the presence of high or low noise. Third, the proposed model can be used for multi-level image segmentation of multi-object image. The proposed algorithm has been applied to natural and synthetic images and has proved to get promising results.

2. Image segmentation procedures

Our segmentation procedure starts with de-noising the image by log Gabor filter. Then the image gradient is used to drive an automatic initialization of a level set contour towards the object. Image forces are balanced with global smoothness constraints to converge stably to a smooth object.

2.1. Image de-noising

Noise is any undesired information that contaminates an image. It appears in images from a variety of sources. The noise can be modelled with either a Gaussian (normal), uniform, or salt –and-pepper (impulse) distribution. The shape of the distribution of Gaussian noise as a function of gray level can be modelled as a normal histogram as shown in Fig. (1).

De-noising of images is typically done by transforming it into some domain where the noise component is more easily identified, a threshlding operation is then applied to remove the noise, and finally the transformation is inverted to reconstruct a noise–free image. If the noise in the image is not normally distributed or non-uniform, the isotropic smoothing Gaussian will not effectively used to remove the noise.

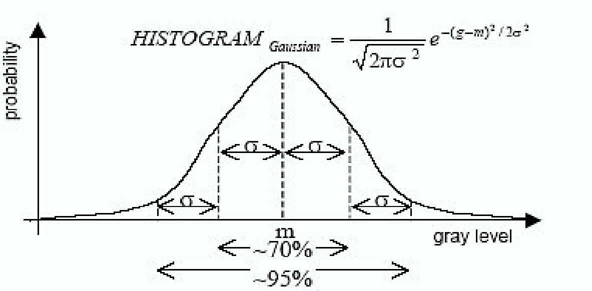

Figure 1 Distribution function of a Gaussian

Field [2] suggests that natural images are better coded by filters that have Gaussian transfer function when viewed on the logarithmic frequency. The log-Gabor function has Gaussian transfer function when viewed on the logarithmic scale and only can be numerically constructed in the spatial domain via the inverse Fourier transform.

$$G\left(\boldsymbol{w}\right) = \boldsymbol{e}^{-\boldsymbol{log}\left(\boldsymbol{w}/\boldsymbol{w}_0\right)^2 / 2\boldsymbol{log}\left(\boldsymbol{k}/\boldsymbol{w}_0\right)^2}, \qquad (6)$$

where $\boldsymbol{w}_0$ represents the center frequency of the filter; *k* determines the bandwidth of the filter in radial direction. Here we follow the approach of Kovesi [4] in which the de-noising process consists of determining a noise threshold at each scale and shrinking the magnitudes of the filter response vectors appropriately while leaving the phase unchanged. If we let $\boldsymbol{I}$ denote the signal and W_n^o and W_n^e denote the odd and even symmetric wavelet at scale *n* given by:

$$W_n = G\left(\boldsymbol{w}\right) * e^{-\left(d\theta\right)^2 / 2\left(\theta_{\sigma}\right)^2} \qquad (7)$$

where, $d\theta$ is the absolute angular distance for each point in the filter matrix from the specified filter orientation, and $\left(\theta_{\sigma} = \pi / \textit{no. of filter orintation}\right)$ is the standard deviation of the angular Gaussian function.

Analysis of the signal $\boldsymbol{I}$ is done by convolving the signal with each quadric pairs of wavelet.

$$\left[r_n, i_n\right] = \left[\boldsymbol{I} \otimes W_n^o, \boldsymbol{I} \otimes W_n^e\right] \qquad (8)$$

The values r_n, i_n are the real and complex values of frequency components. The amplitude of the transform at a given wavelet scale is given by:

$$\boldsymbol{A}_n = \sqrt{\boldsymbol{r}_n^{\,2} + \boldsymbol{i}_n^{\,2}} \qquad (9)$$

De-noising is achieved by determining a threshold at each scale and shrinking the magnitude of the filter $\boldsymbol{A_n}$.

2.2. Image forces

In a deformable model segmentation scheme, the model is driven by image forces and constrained by prior information on the shape of the model. In classical active contour, the image forces are governed by the gradient magnitude and the shape prior is a form of smoothness. While in the Chan and Vese model [1], the contour is evolving by the mean curvature flow. Our active contour model is a combination between the classical active contour based on the gradient of the image and the mean curvature moving technique.

Consider the evolution of the initial set function ϕ such that its zero set tracks the evolving contour at a constant speed *V*. We use the image gradient for moving the contour given by,

$$\frac{\partial \phi}{\partial t} = v \left|\nabla \boldsymbol{I}\right| \tag{10}$$

The initial function ϕ_0 is given by the following equation,

$$\phi_0(x,y) = \begin{cases} -\varepsilon, & (x,y) \in \Omega_0 - \partial\Omega_0 \\ 0 & (x,y) \in \partial\Omega_0 \\ \varepsilon & \Omega\text{-}\Omega_0 \end{cases} \tag{11}$$

Where: Ω_0 is a subset in image domain Ω, and $\partial\Omega_0$ be all the points on the boundaries of Ω_0.

There are two kinds of energies which drive the contour towards the objects. Outside force F_o that pushes the contour towards the object and the other is internal force F_{in} and these forces are related as follows,

$$\frac{\partial \phi}{\partial t} = v\left(F_{in} - F_o\right)\left|\nabla \boldsymbol{I}\right| \tag{12}$$

The image forces need to be balanced with some smoothness constraints; a standard technique is to apply the mean curvature flow to the contour that is the length of the contour and the area inside it. The strength of the smoothing is controlled with the constant factors μ_1 and μ_2:

$$\frac{\partial \phi}{\partial t} = v\left(F_{in} - F_o\right)\left|\nabla \boldsymbol{I}\right| + \mu_1\left(\Delta\phi - div\left(\frac{\nabla\phi}{|\nabla\phi|}\right)\left|\nabla \boldsymbol{I}\right|\right) - \mu_2\left|\nabla \boldsymbol{I}\right| \tag{13}$$

where Δ is the Laplacian operator. Lastly, smoothness constraints are applied to the contour in order to prevent it from leaking into small noise structures in the image that is not part of the target objects. The smoothing behavior is defined by the relationship between the derivative magnitude in the gradient direction and the derivative in the direction of level set. The strength of the smoothness is controlled by constant factor υ. This process is illustrated for natural image in Fig. (2).

$$\frac{\partial \phi}{\partial t} = v\left(F_{in} - F_o\right)\left|\nabla \boldsymbol{I}\right| + \mu_1\left(\Delta\phi - div\left(\frac{\nabla\phi}{|\nabla\phi|}\right)\left|\nabla \boldsymbol{I}\right|\right) - \mu_2\left|\nabla \boldsymbol{I}\right| + \upsilon\left|\nabla \boldsymbol{I}\right| div\left(\frac{\nabla\phi}{|\nabla\phi|}\right) \tag{14}$$

3. Experimental results

The proposed active contour method has been applied to a variety of synthetic and real images in different modalities. In all experiments shown in this section, the level set function is initialized as function ϕ_0 defined by the equation (11) with $\varepsilon = 8$.

The presence of noise in image is filtered by log Gabor filter since such treatment was specially designed to de-noise image while preserving edges. Fig. (3) presents two natural images and one synthetic image with their corresponding segmentation (second row). The outputs from the active contour model with and without edges gradient are shown in third and forth rows in Fig. (3). These images are rich in texture edges.

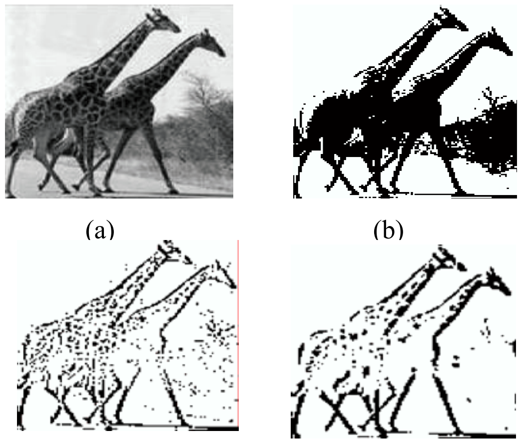

Figure2 (a) Original input image, (b) Segmented image obtained by using image gradient, and (c) after applying smooth constraints, (d) Final segmented image

Fig.(4) illustrates the capability of our method to segment a single object or grouping multi-objects by changing the number of wavelet scales and number of orientation during noise removal and by controlling the number of iteration. On the other hand, the use of our algorithm enables researchers to process and segment images with multiple regions. The result is shown in Fig. (5). Moreover, our model preserves one of the main advantages of the Chan and Vese model that is particularly well-suited for segmentation of images without clearly defined edges as shown in Fig. (6).

Conclusion

We have presented a new hybrid active contour model which combines the classical active contour based on the image gradient and the mean curvature technique. The efficiency of the proposed model is achieved by smoothing the image using log-Gabor filter. Moreover, an addition of smoothness constraint is applied to the evolving contour in order to prevent it from leaking into small noisy structures in the image. Our model can detect objects in non-uniform noisy image, for which the other active contour models are not applicable. In addition, we can detect objects with very small boundaries and segment single objects or grouping multi-objects in the image. We validated our model by various numerical results.

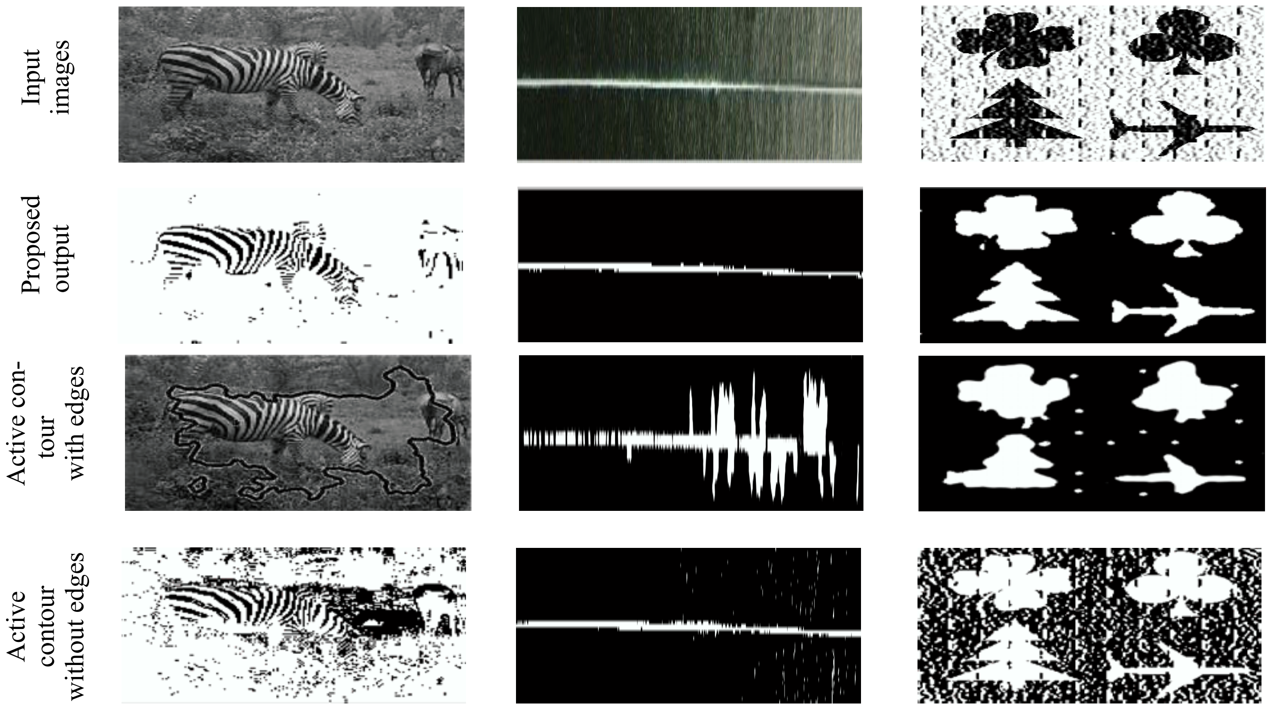

Figure 3. Original noisy images (first row), and its desired output (second row), best active contour obtained with and without edges gradient (third and fourth rows) respectively

References

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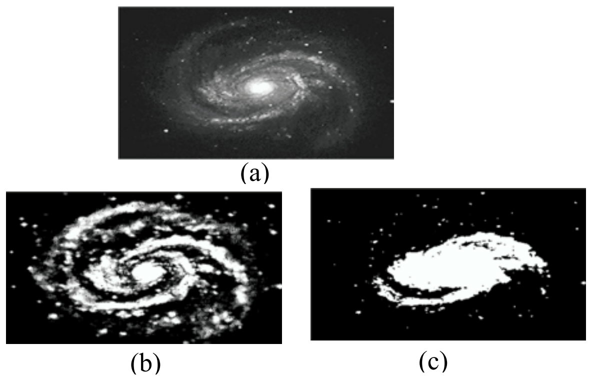

Figure 4. Segmentation of galaxy (a) Original image, (b) and (c) Segmented images with different wavelet scales

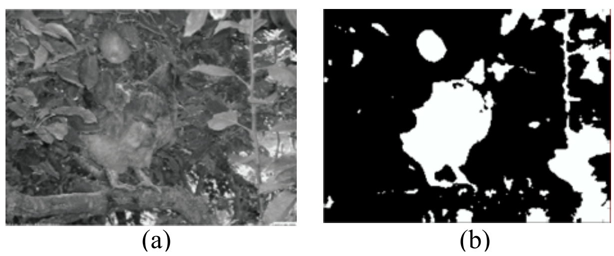

Figure 5. Multiple regions segmentation, (a) Original image, (b) Segmented image

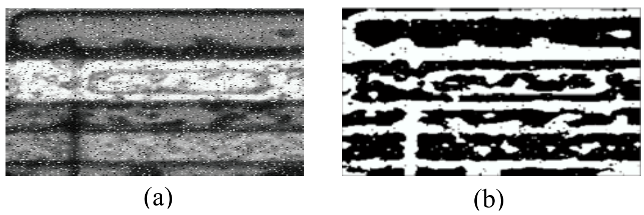

Figure 6. Segmentation of Blurry image (a) Original image, (b) Segmented image